Math 163
Fall 2023
Assessment 4
Dusty Wilson

Name: key

I have hardly ever known a mathematician who was capable of reasoning.

Plato
427 - 347 BC (Greek philosopher)

No work = no credit

1. Warm-ups

(a) (1 point) $\vec{i} \times \vec{k} =$ -j vector

(b) (1 point) $5^2 =$ 25

(c) (1 point) $\vec{i} \cdot \vec{j} =$ 0 scalar

2. (1 point) Based upon Plato's experience (above), how good were mathematicians at thinking/reasoning? Answer using complete English sentences.

In Plato's experience, mathematicians (astrologers) were irrational.

3. (4 points) Find the exact length of the curve $x = 1 + 3t^2$, $y = 4 + 2t^3$, on $0 \le t \le 1$

$$L = \int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$$

$$= \int_0^1 \sqrt{36t^2 + 36t^4}\, dt$$

$$= \int_0^1 6t\sqrt{1+t^2}\, dt$$

Let $u = 1 + t^2$

$du = 2t\, dt$

$u(0) = 1$

$u(1) = 2$

$$= \int_1^2 3\sqrt{u}\, du$$

$$= 2u^{3/2}\Big|_1^2$$

$$= 2\left(2^{3/2} - 1\right)$$

4. (4 points) Find all point(s) on the curve $x = t^3 - 3t$ and $y = t^2 - 10t$ where the tangent is horizontal or vertical.

$$\frac{dy}{dx} = \frac{2t - 10}{3t^2 - 3} = \frac{2(t-5)}{3(t+1)(t-1)}$$

horizonal tangent

$t = 5$

$(110, -25)$

vertical tangent

$t = 1$

$(-2, -9)$

$t = -1$

$(2, 11)$

5. (4 points) Find parametric equations for the tangent line to the curve $x = \ln(t+1)$, $y = t\cos(2t)$, and $z = e^t$ at the point $(0, 0, 1)$

$\leftarrow t = 0$

$$\vec{r}(t) = \langle \ln(t+1),\ t\cos(2t),\ e^{2t} \rangle$$

$$\vec{r}\,'(t) = \left\langle \frac{1}{t+1},\ \cos(2t) + -2t\sin(2t),\ 2e^{2t} \right\rangle \Big|_{t=0} \quad \langle 1, 1, 2 \rangle$$

Tangent line $L(t) = \langle 0, 0, 1 \rangle + t\langle 1, 1, 2 \rangle$

$= \langle t,\ t,\ 2t+1 \rangle$

6. (4 points) Find the unit tangent vector $\vec{T}(t)$ of $\vec{r}(t) = \arctan(t)\vec{i} + 2e^{2t}\vec{j} + 8te^t\vec{k}$ at the point where the parameter $t = 0$.

$$\vec{r}\,'(t) = \left\langle \frac{1}{1+t^2},\ 4e^{2t},\ 8e^t + 8te^t \right\rangle$$

$\Rightarrow \vec{r}\,'(0) = \langle 1, 4, 8 \rangle$

And $|\vec{r}\,'(0)| = \sqrt{1 + 16 + 64} = 9$

$\Rightarrow \vec{T}(0) = \frac{1}{9}\langle 1, 4, 8 \rangle$