100	90s	80s	70s	60s	< 60
0	6	11	3	5	7

Math 163
Fall 2023
Assessment 3
Dusty Wilson

Name: Key

high = 98.2%
$\bar{x}$ = 73.8%
med = 82.1%

He is like the fox, who effaces his tracks in the sand with his tail.

Niels Henrik Abel
1802 - 1829 (Norwegian mathematician

No work = no credit

1. Warm-ups

   (a) (1 point) $\cancel{\vec{i}\times\vec{k}}$ $\vec{k}\times\vec{j}$ = (a) $-\vec{i}$

   (b) (1 point) $\cancel{\frac{2}{0}}$ $\vec{i}\cdot\vec{j}$ = (b) 0

   (c) (1 point) $\cancel{-3^2}$ $2/0$ = (c) undefined

2. (1 point) In the quote above, Abel talks about Gauss' writing style. According to Abel, how easy was it to understand Gauss' work? Answer using complete English sentences.

   Gauss was hard to understand.

3. Answer the following.

   (a) (2 points) Find the equation of a line that goes through the point $(5,4,2)$ and is parallel to the vector $\vec{a} = <6,8,2>$.

   $\vec{r}(t) = \langle 5,4,2\rangle + t\langle 6,8,2\rangle$

   (b) (2 points) Find the equation of the plane that includes the point $(5,5,3)$ and has normal vector $\vec{a} = <8,9,5>$.

   $8(x-5) + 9(y-5) + 5(z-3) = 0$

4. (~~3~~ 4 points) Find the equation of the line in $\mathbb{R}^3$ that goes through the points $A(3,8,4)$ and $B(6,6,1)$. Express your answer in parametric AND symmetric form.

   $\overrightarrow{AB} = \langle 3,-2,-3\rangle \Rightarrow$ line: $\vec{r}(t) = \langle 3,8,4\rangle + t\langle 3,-2,-3\rangle$

   or $\dfrac{x-3}{3} = \dfrac{y-8}{-2} = \dfrac{z-4}{-3}$

5. (~~3~~ 4 points) Find the equation of the plane in $\mathbb{R}^3$ that goes through the points A $(1,1,7)$, B $(7,8,4)$, and C $(2,5,9)$. You do not need to simplify your result.

$\vec{AB} = \langle 6, 7, -3 \rangle$

$\vec{AC} = \langle 1, 4, 2 \rangle$

$\vec{AB} \times \vec{AC} = \langle 26, -15, 17 \rangle$

The plane is

$26(x-1) - 15(y-1) + 17(z-7) = 0$

6. (5 points) Find the equation of the line where the planes $3x + y = 6$ and $y + 5z = 15$ intersect.

$y = 0$: $x = 2$, $z = 3$

so a point is $(2, 0, 3)$

$\vec{n}_1 = \langle 3, 1, 0 \rangle$

$\vec{n}_2 = \langle 0, 1, 5 \rangle$

$\Rightarrow \vec{n}_1 \times \vec{n}_2 = \langle 5, -15, 3 \rangle$

so the line of intersection is:

$\vec{r}(t) = \langle 2, 0, 3 \rangle + t \langle 5, -15, 3 \rangle$

7. Answer the following.

(a) (2 points) Give an example of the equations of two parallel planes.

$x + y + z = 0$ AND $x + y + z = 1$.

(b) (5 points) Give an example of two lines in $\mathbb{R}^3$ that do NOT intersect. Show that the lines do NOT intersect.

$\vec{r}(t) = \langle 1, 1, 1 \rangle + t \langle 1, 2, 3 \rangle$

$\vec{r}(s) = \langle 0, 0, 0 \rangle + s \langle 1, 2, 3 \rangle$

set them equal: $\langle 1, 1, 1 \rangle = s\langle 1, 2, 3 \rangle - t \langle 1, 2, 3 \rangle$

$= (s - t) \langle 1, 2, 3 \rangle$

$\Rightarrow s - t = 1$ AND $2(s-t) = 1$ contradiction.

so the lines don't intersect

8. (5 points) Use your calculator to graph the parametric curve $x = 2 + 3\cos(\theta)$ and $y = 4 + 5\sin(\theta)$ on $\frac{\pi}{2} \le \theta \le 2\pi$. Clearly indicate on the graph the

$\cos\theta = \dfrac{x-2}{3}$ and $\sin\theta = \dfrac{y-4}{5}$

we know

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\dfrac{y-4}{5}\right)^2 + \left(\dfrac{x-2}{3}\right)^2 = 1$