Name: ___key___

No work = no credit
No CAS Calculators

Warm-ups (1 pt each): $-3^2 =$ ___-9___ $\bar{i} \cdot \bar{i} =$ ___1___ $\bar{k} \times \bar{i} =$ ___$\bar{j}$___

1.) (1 pt) A first saw the quote by Bierce (above) on a bumper sticker. In what sense do you think the lottery helps/hurts people? Answer using complete English sentences.

The expected value of the lotto is (generally) well under the cost of a ticket, save your hard earned cash!

2.) (8 pts) Consider $\bar{a} = \langle -8, 0, 2 \rangle$ and $\bar{b} = \langle 1, 3, -3 \rangle$, find the following.

a.) $\bar{a} \cdot \bar{b} = -8 + 0 - 6 = -14$

b.) The angle between vectors $\bar{a}$ and $\bar{b}$. Give your answer to one decimal place.

$$\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{-14}{\sqrt{68}\sqrt{19}} \right)$$

$$= 112.9° \quad \text{or}$$

$$= 2.0 \text{ rad.}$$

c.) $|\bar{a}|^2 = \bar{a} \cdot \bar{a}$

$$= 68$$

d.) $\text{proj}_{\bar{a}} \bar{b} = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}|^2} \bar{a}$

$$= \frac{-14}{68} \langle -8, 0, 2 \rangle$$

3.) (4 pts) A sled is pulled along a level path through snow by a rope. A 20-lb force acting at an angle of 35 degrees above the horizontal moves the sled 60 feet. Find the work done by the force (in foot-pounds). Give your answer to one decimal place.
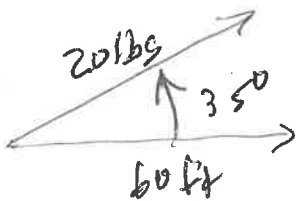
$$W = \bar{F} \cdot \bar{D} = |\bar{F}||\bar{D}| \cos\theta$$

$$= 20(60)\cos 35°$$

$$\approx 983.0 \text{ ft-lbs.}$$

4.) (4 pts) What is the relationship between perpendicular vectors and:

a.) The dot product

the dot product of ⊥ vecs is 0.

b.) The cross product

either ok { (a) the cross product creates ⊥ vectors.
(b) " " " of ⊥ vecs has max magnitude*

5.) (4 pts) Consider $\vec{a} = -8\vec{i} + 2\vec{k}$ and $\vec{b} = \vec{i} + 3\vec{j} - 3\vec{k}$, find the following.

a.) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8 & 0 & 2 \\ 1 & 3 & -3 \end{vmatrix}$
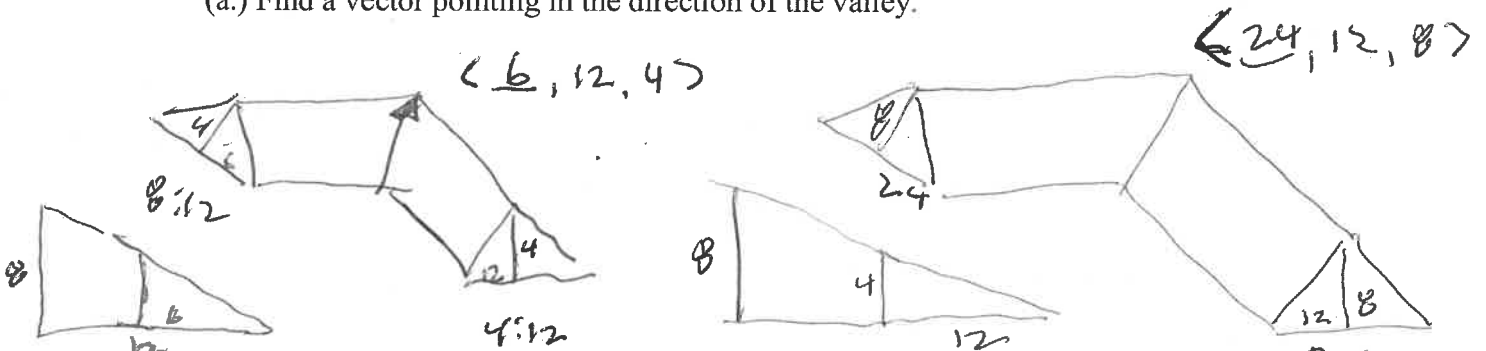
$= \langle -6, -22, -24 \rangle$

b.) Find the area of the parallelogram that has the two adjacent sides $\vec{a}$ and $\vec{b}$. Give the exact answer.

$area = |\langle -6, -22, -24 \rangle| = \sqrt{6^2 + 22^2 + 24^2} = \sqrt{1096}$

6.) (4 pts) Two roof planes meet at a 90-degree angle. The first plane has a 4:12 pitch. The second plane has an 8:12 pitch.

(a.) Find a vector pointing in the direction of the valley.

$\langle 6, 12, 4 \rangle$

$\langle 24, 12, 8 \rangle$

(b.) Find the angle between the valley and a line going straight up the first roof plane.

Angle between $\langle 0, 12, 4 \rangle$ and $\langle 6, 12, 4 \rangle$
      $\vec{a}$ $\vec{b}$

$\theta = \cos^{-1}\left(\frac{160}{\sqrt{160} \cdot \sqrt{196}} \right) = 25.4°$