Math 163
Fall 2023
Assessment 6
Dusty Wilson

Name: ______________________________

He knew he couldn't tell stories, that he always included extraneous details and tangents that interested only him.
John Green
1977 - present (American author)

No work = no credit

1. (3 points) Warm-ups

   (a) $\frac{d}{dx}\sin(x^2) =$ (b) $\frac{\partial}{\partial x}\arctan(x) =$ (c) $\frac{\partial}{\partial y} y\sin(x^2) =$

2. (1 point) What does it mean to "go off on a tangent"? Answer using complete English sentences.

3. (8 points) Consider $z = 3x + 4y^2$

   (a) (4 points) Find the tangent plane (linear approximation) at the point $A(2, 3)$

   (b) (4 points) If (x, y) changes from $A(2, 3)$ to $B(1.7, 3.1)$ compare the values of Δz and dz

4. (4 points) If $z = 3x\sin(2y)$, $x = e^{rt}$, and $y = r\ln rt$, find $\frac{\partial z}{\partial t}$. Your result may include the variables: x, y, r, and t

5. (14 points) Consider the function $g(x, y) = 2x^3 - 3xy^4$

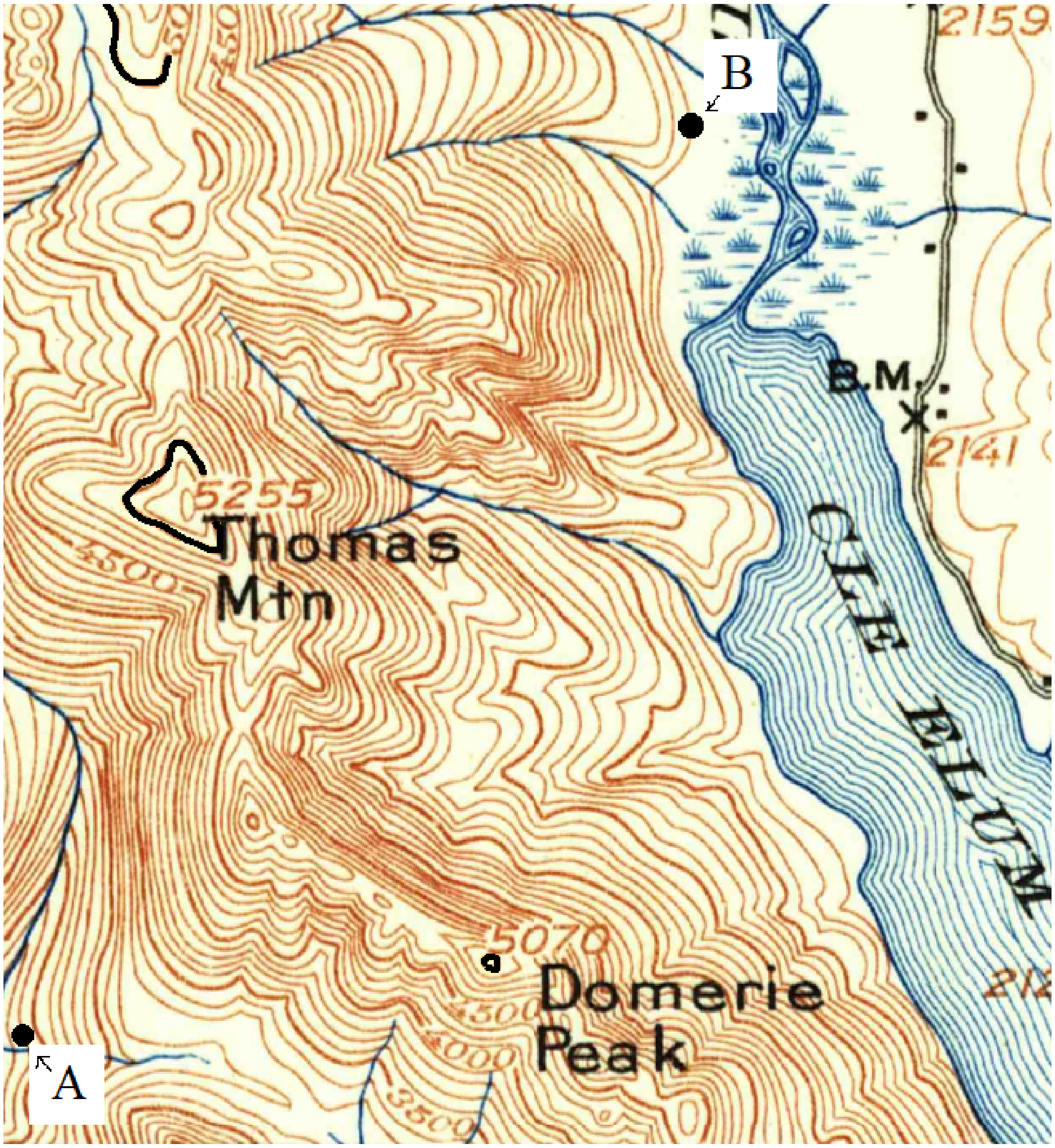
   (a) (4 points) Find the gradient at point $A(2, 1)$

   (b) (4 points) Find and interpret the (i.) direction and (ii.) magnitude of the gradient at point A. Your answers should include both vectors/numbers and a written interpretation.

   (c) (2 points) At point A, in what direction $\langle x, y\rangle$ should we travel if we want our height on g to remain constant (NOT change)? Hint: There are multiple correct answers.

   (d) (4 points) Find the directional derivative of g at point A in the direction of the vector $\vec{v} = \langle 8, 15\rangle$

6. (4 points) Consider the contour plot (topographical map) of the mountains near Snoqualmie Pass where $z = f(x, y)$ gives the altitude in feet at a point (x, y) where x and y have the traditional orientation. The solid black line(s) show level curves at 5,000 feet.



(a) (1 point) On the contour plot, clearly sketch at least 5 possible gradient vectors near Lake Cle Elum

(b) (1 point) On the contour plot, clearly mark with a $\diamondsuit$ the point(s) of the level curve near Thomas Mtn where $f(x, y) = 5000$ at which $f_x = 0$ and $f_y < 0$

(c) (1 point) On the contour plot, clearly mark with a $\heartsuit$ the point(s) of the level curve near Thomas Mtn where $f(x, y) = 5000$ at which the slope is smallest ($|\nabla f|$ is small).

(d) (1 point) Beginning at point B, clearly sketch the path of steepest ascent.