

Name: \_\_\_\_\_

**Math 163**  
**Fall 2023**  
**Assessment 5**  
**Dusty Wilson**

**No work = no credit**

*If there is anything that can bind the mind of man to this dreary exile of our earthly home and can reconcile us with our fate so that one can enjoy living,—then it is verily the enjoyment of the mathematical sciences and astronomy.*

Johannes Kepler

1571 - 1630 BC (Swiss astronomer)

1. (3 points) Warm-ups

(a)  $\frac{d}{dx} \tan(x) =$

(b)  $\frac{\partial}{\partial x} \arctan(x) =$

(c)  $\frac{\partial}{\partial x} \sqrt{y} =$

2. (1 point) What did Kepler look to (above) that brought him pleasure? Answer using complete English sentences.

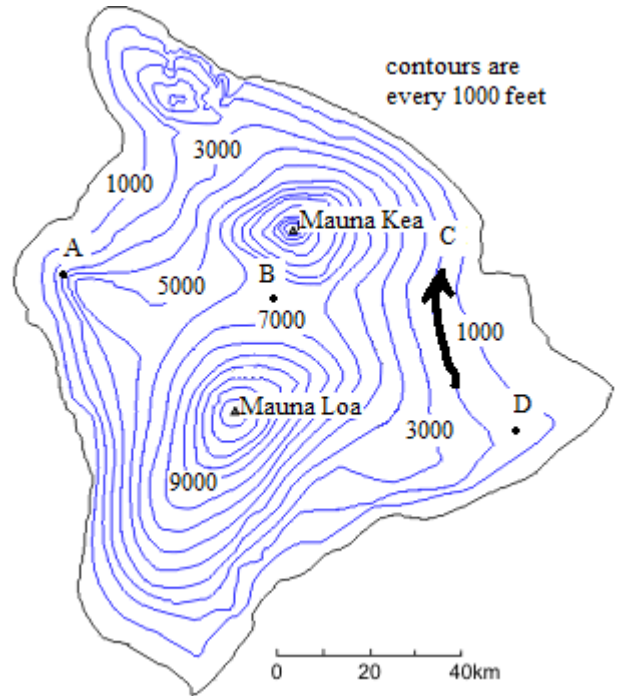
3. (4 points) Consider  $f(x, y) = 2x^2 \sin(y^4) + 5x - 6y + 7$

(a) (2 points) Find  $f_x(x, y)$

(b) (2 points) Find  $\frac{\partial^2 f}{\partial x \partial y}$

4. (4 points) Use the contour plot of the Island of Hawai'i to answer the following:

- (a) (1 point) Estimate the height of the highest point on Mauna Kea
- (b) (1 point) At which point is the terrain steepest? A, B, or D? Why?
- (c) (1 point) Suppose you were hiking along path C. What can you say about your altitude?
- (d) (1 point) Explain what is happening to the altitude (height) around point B.



5. (19 points) Consider  $\vec{r}(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$  on  $-\frac{\pi}{2} < t < \frac{\pi}{2}$

- (a) (1 point) If  $\vec{r}(t)$  includes  $(1, 0, 0)$ , what is  $t$ ? Use this value of  $t$  in subsequent parts of this question.
- (b) (4 points) Find and fully simplify  $|\vec{r}''(t)|$
- (c) (4 points) Find the curvature of  $\vec{r}(t)$  at the  $t$ -value found in (a).

Reminder from the previous page:  $\vec{r}(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$  on  $-\frac{\pi}{2} < t < \frac{\pi}{2}$

(d) (4 points) Find the tangential and normal components of acceleration at the  $t$ -value found in (a).

(e) (6 points) Find the tangential, normal, and binormal vectors at the  $t$ -value found in (a).