Assessment 8
Dusty Wilson
Math 163

No work = no credit

Name: Key.

With the exception of the geometric series, there does not exist in all of mathematics a single infinite series whose sum has been determined rigorously.

Niels Henrik Abel
1802-1829 (Norse mathematician)

Warm-ups (1 pt each): $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$ $\qquad \sum_{n=0}^{\infty} 1 = \infty$ $\qquad \sum_{n=0}^{\infty} 2\left(\frac{1}{3}\right)^n = \frac{2}{1-\frac{1}{3}} = 3$

1.) (1 pt) In the quote, Abel indicates that there was only one series whose sum was well established. Do you feel like the power series discussed in class have been studied rigorously? Why or why not? Answer using complete English sentences.

I feel like we have done a good job w/ the big picture & usefulness, but spent little time on rigor.

2.) (5 pts) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

using the ratio test,

solve $\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n x^n} \right| < 1$

$\Rightarrow |x| \lim_{n\to\infty} \frac{n^2}{(n+1)^2} < 1$

$\Rightarrow |x| < 1$

so $R = 1$

3.) (5 pts) Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$

using the ratio test

solve $\lim_{n\to\infty} \left| \frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}} \cdot \frac{5^n \sqrt{n}}{(2x-1)^n} \right| < 1$

$\Rightarrow \frac{|2x-1|}{5} \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} < 1$

$\Rightarrow \frac{|2x-1|}{5} < 1$

$\Rightarrow |2x-1| < 5$

$\Rightarrow \left|x - \frac{1}{2}\right| < \frac{5}{2}$

so $R = \frac{5}{2}$

4.) (1 pt) What is a power series?

A power series is an infinitely long polynomial. More specifically, it is a fct of the form $f(x)=\sum_{n=0}^{\infty} c_n x^n$ for some coefficients $c_0, c_1, c_2, \ldots$ & exists where the series converges.

5.) (5 pts) Find the Maclaurin series for $\cos(2x)$. Please show the process/derivation although you may certainly check by modifying a known power series.

$$f(x) = \cos 2x \Big|_{x=0} \quad 1$$

$$f'(x) = -2\sin 2x \Big|_{x=0} \quad 0$$

$$f''(x) = -4\cos 2x \Big|_{x=0} \quad -2^2$$

$$f^{(3)}(x) = 8\sin 2x \Big|_{x=0} \quad 0$$

$$f^{(4)}(x) = 16\cos 2x \Big|_{x=0} \quad 2^4$$

$$\cos 2x = 1 - \frac{2^2}{2!}x^2 + \frac{2^4}{4!}x^4 - \ldots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$$

6.) (5 pts) Find a Maclaurin series for $h(x) = x^3 e^{x^2}$ by modifying a known power series.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\Rightarrow e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$$

$$\Rightarrow x^3 e^{x^2} = \sum_{n=0}^{\infty} \frac{x^{2n+3}}{n!}$$