A5

Saturday, February 18, 2023 3:20 PM

Assessment 5
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Math 163

Name: key

There still remain three studies suitable for free man. Arithmetic is one of them.

Plato
427 – 347 BC (Greek philosopher)

No work = no credit

Warm-ups (1 pt each): $\frac{\partial}{\partial x}x^2 = 2x$ $\quad \frac{\partial}{\partial x}y^2 = 0$ $\quad \frac{\partial}{\partial x}\ln(xy) = \frac{y}{xy} = \frac{1}{x}$

1.) (1 pt) Plato said that arithmetic is worthy of study along with two other topics. What do you think were the other topics worth studying? (I don't know, I'm just curious what you think). Answer using complete English sentences.

Perhaps Plato referred to philosophy and logic? ① Arithmetic ② geometry, And ③ astronomy.

2.) (5 pts) Find the curvature of the helix $\vec{r}(t) = \langle 2\cos t, 2\sin t, 3t \rangle$.

$\vec{r}'(t) = \langle -2\sin t, 2\cos t, 3 \rangle$

$\vec{r}''(t) = \langle -2\cos t, -2\sin t, 0 \rangle$

$\vec{r}' \times \vec{r}'' = \langle 6\sin t, -6\cos t, 4 \rangle$

And $|\vec{r}'| = \sqrt{4\sin^2 t + 4\cos^2 t + 9} = \sqrt{13}$

And $|\vec{r}' \times \vec{r}''| = \sqrt{36\sin^2 t + 36\cos^2 t + 16} = \sqrt{52}$

$\Rightarrow k = \frac{\sqrt{52}}{(\sqrt{13})^3}$

3.) (5 pts) Find the tangent, normal, and binormal vectors of the curve $\vec{r}(t)=\left\langle \frac{t^2}{2}, 4-3t, 1\right\rangle$ at the point $(2,-2,1)$.

$\boxed{t=2}$

$\vec{r}'(t) = \langle t, -3, 0\rangle$

$\Rightarrow |\vec{r}'(t)| = \sqrt{t^2+9}$

$\Rightarrow \vec{T}(t) = \frac{1}{\sqrt{t^2+9}}\langle t, -3, 0\rangle \Big|_{t=2} \quad \vec{T} = \frac{1}{\sqrt{13}}\langle 2, -3, 0\rangle$

And $\vec{T}'(t) = -\frac{1}{\cancel{2}}(t^2+9)^{-3/2}\cdot \cancel{2}t\,\langle t, -3, 0\rangle + \frac{1}{\sqrt{t^2+9}}\langle 1, 0, 0\rangle$

$= \frac{-t}{(t^2+9)^{3/2}}\langle t, -3, 0\rangle + \left\langle \frac{1}{\sqrt{t^2+9}}, 0, 0\right\rangle$

$\Rightarrow \vec{T}'(2) = \frac{-2}{13^{3/2}}\langle 2, -3, 0\rangle + \left\langle \frac{13}{\sqrt{13}^3}, 0, 0\right\rangle$

$= \left\langle \frac{9}{13^{3/2}}, \frac{6}{13^{3/2}}, 0\right\rangle$

$\Rightarrow |\vec{T}'(2)| = \sqrt{\frac{81}{13^3} + \frac{36}{13^3} + 0} = \frac{\sqrt{117}}{13^{3/2}}$

$$\Rightarrow |\vec{T}'(2)| = \sqrt{\frac{81}{13^3} + \frac{36}{13^3} + 0} = \frac{\sqrt{117}}{13^{3/2}}$$

$$\Rightarrow \vec{N}(2) = \frac{\vec{T}'(2)}{|\vec{T}'(2)|} = \frac{1}{\sqrt{117}}\langle 9, 6, 0\rangle$$

AND $\vec{B}(2) = \vec{T}(2) \times \vec{N}(2)$

$$= \frac{1}{\sqrt{13}}\langle 2, -3, 0\rangle \times \frac{1}{\sqrt{117}}\langle 9, 6, 0\rangle$$

$$= \frac{1}{\sqrt{1521}}\langle 0, 0, 39\rangle$$

$$= \langle 0, 0, 1\rangle$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 0 \\ 9 & 6 & 0 \end{vmatrix}$$

$= \langle 0, 0, 39\rangle$

4.) (5 pts) Find the tangential and normal components of the acceleration if $\vec{r}(t) = \left\langle \frac{t^2}{2}, 4-3t, 1 \right\rangle$ at the point $(2,-2,1)$.

$\boxed{t = 2}$

Hint: You can check by using combining the results of (3.) and (4.) and verifying that $\vec{a} = a_T\vec{T} + a_N\vec{N}$.

$\vec{r}'(t) = \langle t, -3, 0\rangle \big|_{t=2} \quad \langle 2, -3, 0\rangle$

$\vec{r}''(t) = \langle 1, 0, 0\rangle$

$$a_T = \frac{\vec{r}' \cdot \vec{r}''}{|\vec{r}'|} = \frac{2}{\sqrt{13}}$$

$$a_N = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|} = \frac{3}{\sqrt{13}}$$

scratch.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$\langle 0, 0, 3\rangle$

$|\vec{r}'|$ $\sqrt{13}$

check:

$$a_T \vec{T} + a_N \vec{N} = \frac{2}{\sqrt{13}} \frac{1}{\sqrt{13}}\langle 2,-3,0\rangle + \frac{3}{\sqrt{13}} \frac{1}{\sqrt{117}}\langle 9,6,0\rangle$$

$$= \frac{2}{13}\langle 2,-3,0\rangle + \frac{\cancel{3}^{\,1}}{\cancel{39}_{13}}\langle 9,6,0\rangle$$

$$= \langle 1,0,0\rangle \checkmark$$

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