Assessment 4
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Math 163

Name: Key.

No work = no credit

I have hardly ever known a mathematician who was capable of reasoning.

Plato
427 – 347BC (Greek philosopher)

Warm-ups (1 pt each): $\vec{i} \times \vec{k} =$ $-\vec{j}$ $\quad \vec{i} \cdot \vec{k} =$ 0 $\quad \vec{i} \cdot \vec{i} =$ 1

1.) (1 pt) Based upon Plato's experience (above), how good were mathematicians at thinking/reasoning? Answer using complete English sentences.

Mathematicians were Not good @ reasoning.

2.) (5 pts) Find the equation of the tangent line to the cycloid $x = 3(\theta - \sin\theta)$ and $y = 3(1 - \cos\theta)$ at the point where $\theta = \frac{\pi}{3}$.

need ① point $\rightarrow x = 3\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$ & $y = 3\left(1 - \frac{1}{2}\right)$

② slope.

$\frac{dy}{d\theta}$ $\quad$ $\frac{dx}{d\theta}$

$\frac{dy}{d\theta} = 3\sin\theta \qquad \frac{dx}{d\theta} = 3 - 3\cos\theta$

$$\Rightarrow \frac{dy}{dx} = \left.\frac{3\sin\theta}{3 - 3\cos\theta}\right|_{\theta = \pi/3} \frac{3 \cdot \frac{\sqrt{3}}{2}}{3\left(1 - \frac{1}{2}\right)} = \frac{\frac{\cancel{3}\sqrt{3}}{\cancel{2}}}{\frac{\cancel{3}}{\cancel{2}}} = \sqrt{3} \text{ (slope)}$$

tangent line: $y - \frac{3}{2} = \sqrt{3}\left(x - \left(\pi - \frac{3\sqrt{3}}{2}\right)\right)$

3.) (5 pts) Use the calculus of parametric curves to derive the circumference of a circle with radius one. To do this, first parametrize the unit circle making sure to include limits of ~~integration~~. Then find the length of the curve.

your parameter.

1 parametrize

$x = \cos t$

$y = \sin t$

on $0 \le t \le 2\pi$.

2 $L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1}\, dt$

$= \int_0^{2\pi} 1\, dt$

$= 2\pi.$

4.) (5 pts) If $\vec{r}(t) = \langle 1+t^3, te^{-t}, \sin 2t \rangle$, find the unit tangent vector at the point $(1,0,0)$

1 find the point/time.

$t = 0.$

2 find $\vec{r}\,'(t)$

$\vec{r}\,'(t) = \langle 3t^2, e^{-t} - te^{-t}, 2\cos 2t \rangle$

and $\vec{r}\,'(0) = \langle 0, 1, 2 \rangle$

3 find $|\vec{r}\,'(0)| = \sqrt{0+1+4} = \sqrt{5}$

4 $\vec{T}(0) = \frac{1}{\sqrt{5}} \langle 0, 1, 2 \rangle.$