Assessment 7 (10 or 11 am)
Dusty Wilson
Math 220

No work = no credit

Name (first & last): key

The infinite we shall do right away.
The finite may take a little longer.

Stanislaw Ulam
1909-1984 (Polish mathematician)

Warm-ups (1 pt each): $\underset{B\leftarrow C}{P}\,\underset{C\leftarrow B}{P} = I$ $\quad A\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}12\\8\end{bmatrix}: \lambda = 4$ $\quad I_{3x3}: \lambda = 1$

1.) (1 pt) According to Ulam (above), what is it that takes longer than the infinite? Please answer using complete sentences.

The finite is harder than the infinite

2.) (4 pts) Find the eigenspace of $A=\begin{bmatrix}4 & 2 & 3\\ -1 & 1 & -3\\ 2 & 4 & 9\end{bmatrix}$ corresponding to the eigenvalue $\lambda = 3$.

$$A - 3I = \begin{bmatrix}1 & 2 & 3\\ -1 & -2 & -3\\ 2 & 4 & 6\end{bmatrix}$$

$$\sim \begin{bmatrix}1 & 2 & 3\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$E_{\lambda=3} = \text{span}\left\{ \begin{bmatrix}-2\\1\\0\end{bmatrix}, \begin{bmatrix}-3\\0\\1\end{bmatrix} \right\}$$

3.) (4 pts) Find the eigenvalues (and multiplicity) of $A = \begin{bmatrix} 0 & 0 & 0 & 5 \\ 0 & 0 & 8 & -4 \\ 0 & 1 & 7 & 0 \\ 1 & 2 & -5 & 3 \end{bmatrix}$.

Everyone was given full credit for this question.

~~$\lambda = 1$ (mult 2), 5, 8~~

The reversed diagonal makes this question extremely difficult. It requires solving a quartic equation. Numerically, the eigenvalues are about 2.33, 8.23, and -0.19+/-1.5i

4.) (4 pts) Find the eigenvalues (and multiplicity) of $A = \begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$. Use algebraic methods (no calculator).

solve $0 = \begin{vmatrix} 6-\lambda & -2 & 0 \\ -2 & 9-\lambda & 0 \\ 5 & 8 & 3-\lambda \end{vmatrix}$

$= (3-\lambda)\begin{vmatrix} 6-\lambda & -2 \\ -2 & 9-\lambda \end{vmatrix}$

$= (3-\lambda)\big[\underbrace{(6-\lambda)(9-\lambda)-4}_{\lambda^2 - 15\lambda + 50}\big]$

$0 = (3-\lambda)(\lambda-10)(\lambda-5)$

$\lambda = 3, 5, 10$

w/ mult. 1 for all.

5.) (4 pts) Let $B = \left\{ \begin{bmatrix} 7 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ and $C = \left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$ be bases for $\mathbb{R}^2$.

a.) Find the change-of-coordinates matrix from C to B.

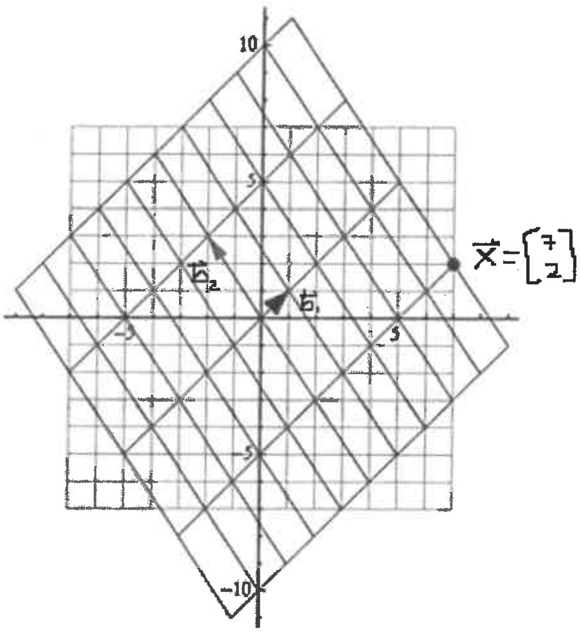
$\underset{B \leftarrow C}{P}$

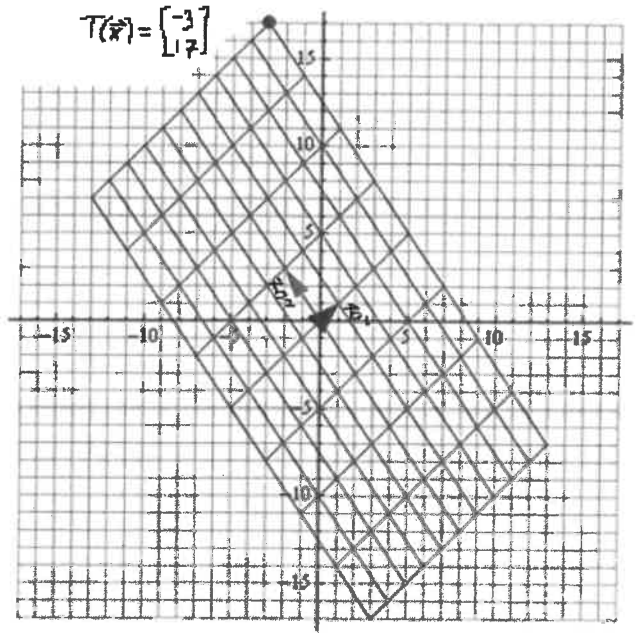
$\left[\begin{array}{cc|cc} 7 & 2 & 4 & 5 \\ -2 & -1 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 2 & 3 \\ 0 & 1 & -5 & -8 \end{array}\right]$ so $\underset{B \leftarrow C}{P} = \begin{bmatrix} 2 & 3 \\ -5 & -8 \end{bmatrix}$

b.) If $[\vec{x}]_B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, find $[\vec{x}]_C$

$\underset{C \leftarrow B}{P} = -\begin{bmatrix} -8 & -3 \\ 5 & 2 \end{bmatrix}$ and $[\vec{x}]_C = \begin{bmatrix} 8 & 3 \\ -5 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 14 \\ -9 \end{bmatrix}$

6.) (4 pts) The left graph shows $\bar{x}$ with respect to the standard and B-coordinates. The right graph shows $T(\bar{x}) = A\bar{x}$ with respect to the standard and B-coordinates. The transformation matrix with respect to the B-coordinates is $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.





a.) Find $[\bar{x}]_B$ $= \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$\vec{b}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \vec{b}_2 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

b.) Find $[T(\bar{x})]_B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

c.) Find the change of coordinate matrix P_B $= \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

d.) Find A, the transformation matrix with respect to the standard basis (Note: There are at least two ways to find A)..

$$A = P B P^{-1} = \begin{bmatrix} -\frac{1}{5} & -\frac{4}{5} \\ \frac{9}{5} & \frac{11}{5} \end{bmatrix}$$