100	90's	80's	70's	60's	< 60
2	10	7	14	9	7

Assessment 3 (10 or 11 am)
Dusty Wilson
Math 220

Name (first & last): Key

No work = no credit

$\bar{x} = 75.67$

med = 75

Biographical history, as taught in our public schools, is still largely a history of boneheads: ridiculous kings and queens, paranoid political leaders, compulsive voyagers, ignorant generals -- the flotsam and jetsam of historical currents. The men [and women] *who radically altered history, the great scientists and mathematicians, are seldom mentioned, if at all.*

Martin Gardner
1914-2010 (American mathematician)

$\underset{2\times1}{\begin{bmatrix}1\\0\end{bmatrix}}\underset{1\times2}{\begin{bmatrix}0 & 1\end{bmatrix}}$

Warm-ups (1 pt each): $\vec{e}_1\vec{e}_2^{\,T} = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$ $\quad \vec{e}_2^{\,T}\vec{e}_1 = \begin{bmatrix}0\end{bmatrix}$ $\quad AA^{-1} = I$

1.) (1 pt) According to Gardner (above), who should receive more focus in history classes?

Those who changed history thru science and math merit more attention.

2.) (8 pts) Consider the matrices

$$A=\begin{bmatrix}4 & -3 & 2\\2 & 0 & -1\end{bmatrix},\ B=\begin{bmatrix}-3 & -4 & 1\\1 & -5 & 7\end{bmatrix},\text{ and } C=\begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}$$

Compute $A+2B$, BC, and CB. If an expression is undefined, explain why.

$$A+2B = \begin{bmatrix}4 & -3 & 2\\2 & 0 & -1\end{bmatrix} + 2\begin{bmatrix}-3 & -4 & 1\\1 & -5 & 7\end{bmatrix} = \begin{bmatrix}-2 & -11 & 4\\4 & -10 & 13\end{bmatrix}$$

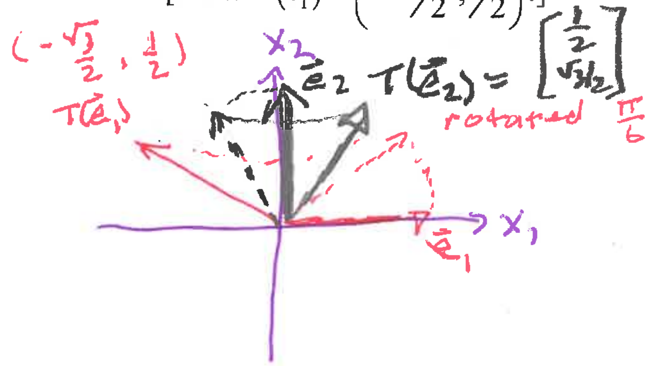
BC is undefined $(2\times3)\cdot(2\times2)$

must match

$$CB = \begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}\begin{bmatrix}-3 & -4 & 1\\1 & -5 & 7\end{bmatrix} = \begin{bmatrix}-5 & 6 & -13\\-5 & -32 & 31\end{bmatrix}$$

3.) (8 pts) $T:\mathbb{R}^2 \to \mathbb{R}^2$ first rotates points through $\pi/6$ radians and then reflects the points through the vertical x_2-axis. Assume that T is a linear transformation. Find the standard matrix A.
of T. [Hint: $T(\vec{e}_1) = \left(-\sqrt{3}/2, 1/2\right)$.]

$T(\vec{x}) = A\vec{x}$



-1 if

-2 no rotated thru x_1-axis.

$$T(\vec{x}) = A\vec{x} \quad w/ \quad A = \begin{bmatrix} | & | \\ T(\vec{e}_1) & T(\vec{e}_2) \\ | & | \end{bmatrix}$$

$$= \begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ \frac{1}{2} & \sqrt{3}/2 \end{bmatrix}$$

4.) (8 pts) Use the **matrix inverse** to solve $A\vec{x} = \vec{b}$ where $A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$.

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right]$$

$$= [I | A^{-1}]$$

$$\vec{x} = A^{-1}\vec{b} \Rightarrow \vec{x} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$$

-1 if A^{-1} $(\cancel{3}\times\cancel{3})$

-3 for $\begin{bmatrix} 7 \\ -9 \\ 3 \end{bmatrix}$

5.) (6 pts) True or False (circle one). Justify your answer.

a.) (T or F) A linear transformation $T:\mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effect on the columns of the $n\times n$ identity matrix.

$T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} | & & | \\ T(\vec{e}_1) & \cdots & T(\vec{e}_n) \\ | & & | \end{bmatrix}$

and $\vec{e}_1, \ldots, \vec{e}_n$ are the cols of the $n \times n$ identity matrix.

b.) (T or F) $(AB)^T = A^TB^T$

$(AB)^T = B^TA^T$

c.) (T or F) If A and B are $n\times n$ and invertible than $A^{-1}B^{-1}$ is the inverse of AB.

$(AB)^{-1} = B^{-1}A^{-1}$

6.) (~~5~~ 4 pts) Complete the following proof.

Claim: Let $T:\mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then T is one-to-one if and only if the equation $T(\vec{x}) = \vec{0}$ has only the trivial solution.

Proof.

($\Rightarrow$) Assume T is one-to-one.
we know $T(\vec{0}) = \vec{0}$ (since T is a L.T.)
$\Rightarrow T(\vec{x}) = \vec{0}$ has a solution and it must be unique since T is one-to-one.

($\Leftarrow$) Assume $T(\vec{x}) = \vec{0}$ has only the trivial solution.
Suppose T is not one-to-one.
$\Rightarrow$ there exists $\vec{u} \neq \vec{v} \in \mathbb{R}^n$ and $\vec{b} \in \mathbb{R}^m$ such that $T(\vec{u}) = \vec{b}$ and $T(\vec{v}) = \vec{b}$
$\Rightarrow T(\vec{u} - \vec{v}) = T(\vec{u}) - T(\vec{v}) = \vec{b} - \vec{b} = \vec{0}$
but $T(\vec{x}) = \vec{0}$ has only the trivial soln so $\vec{u} = \vec{v}$,
$\Rightarrow \Leftarrow$ (contradiction).
$\Rightarrow T$ is one-to-one.

$\therefore T$ is one-to-one iff $T(\vec{x}) = \vec{0}$ has only the trivial solution.