A3

Thursday, February 9, 2023 6:27 PM

Assessment 3
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Math 220

No work = no credit

Name: Key

The lectures were difficult, and only serious students could understand them. ... During his lectures he did not bother about the order of equations on the blackboard, nor about his personal appearance.

Andrei Andreyevich Markov
1856 - 1922 (Russian mathematician)

$\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Warm-ups (1 pt each):
Note: Assume $\vec{e}_1, \vec{e}_2 \in \mathbb{R}^2$

$\vec{e}_2 + \vec{e}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\qquad \frac{6}{0} =$ undefined $\qquad \vec{e}_1^{\,T}\vec{e}_2 = [0]$ or "0"

1.) (1 pt) The quote above describes Markov as a teacher. Do you think Markov would have gotten good reviews on RateMyProfessor? Answer using complete English sentences.

Markov was brilliant, but would have looked bad in the online reviews.

2.) (5 pts) Calculate the determinant of $A = \begin{bmatrix} 1 & -1 & -2 & 0 \\ 0 & 1 & 5 & 4 \\ 4 & -2 & -3 & 4 \\ -1 & 2 & 7 & 5 \end{bmatrix}$ by hand (it is okay to check with a calculator). Is matrix A invertible? Why or why not?

Ans. $\det A = -5$

$\Rightarrow$ A is invertible since $\det A \neq 0$.

$$\det A = +1\begin{vmatrix} 1 & 5 & 4 \\ -2 & -3 & 4 \\ 2 & 7 & 5 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 5 & 4 \\ 4 & -3 & 4 \\ -1 & 7 & 5 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 1 & 4 \\ 4 & -2 & 4 \\ -1 & 2 & 5 \end{vmatrix} - 0$$

$$= \left(+1\begin{vmatrix} -3 & 4 \\ 7 & 5 \end{vmatrix} - 5\begin{vmatrix} -2 & 4 \\ 2 & 5 \end{vmatrix} + 4\begin{vmatrix} -2 & -3 \\ 2 & 7 \end{vmatrix}\right)$$

$$= \left(-15 - 28 - 5(-10-8) + 4(-14+6)\right)$$

$$= 15 + 1\left(+0 - 5\underbrace{\begin{vmatrix} 4 & 4 \\ -1 & 5 \end{vmatrix}}_{24} + 4\underbrace{\begin{vmatrix} 4 & -3 \\ -1 & 7 \end{vmatrix}}_{25}\right) - 2\left(+0 - 1\underbrace{\begin{vmatrix} 4 & 4 \\ -1 & 5 \end{vmatrix}}_{24} + 4\underbrace{\begin{vmatrix} 4 & -2 \\ -1 & 2 \end{vmatrix}}_{6}\right)$$

$$= 15 - 20 + 0.$$

$$= -5.$$

3.) (10 pts) Answer the following:

a.) Calculate $\begin{bmatrix} -1 & 2 \\ 5 & 4 \\ 2 & -3 \end{bmatrix} - 2\begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} -9 & 6 \\ 11 & 4 \\ -4 & -9 \end{bmatrix}$

b.) Calculate $\underset{3\times 2}{\begin{bmatrix} -1 & 2 \\ 5 & 4 \\ 2 & -3 \end{bmatrix}} \underset{2\times 4}{\begin{bmatrix} 3 & 5 & -5 & 1 \\ -1 & 4 & 3 & 1 \end{bmatrix}} = \underset{3\times 4}{\begin{bmatrix} -5 & 3 & 11 & 1 \\ 11 & 41 & 13 & 9 \\ 9 & -2 & -19 & -1 \end{bmatrix}}$

c.) Is $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ singular? If possible, find A^{-1}.

$\det(A) = 0.$

$\Rightarrow$ A is singular.

And A^{-1} d.n.e.

d.) Explain the process for finding the inverse of an n x n matrix A.

$\text{rref}([A \mid I_{n\times n}]) = [I \mid A^{-1}] \Rightarrow A^{-1}$ is...

OR

$= [\text{not } I \mid \text{stuff}] \Rightarrow$ A is singular.

e.) How many rows does B have if BC is a 3x4 matrix?

$B_{3\times p}\, C_{p\times 4} = (BC)_{3\times 4}$

B has 3 rows!! :)

4.) (7 pts) For the matrix A_{nxn}, there are at least 13 statements equivalent to, "A is invertible." List at least seven of them. List more for extra credit.

i.) A is invertible.	vi.) the lin trans. $\vec{x} \mapsto A\vec{x}$ is 1-1.
ii.) $A \sim I_{N \times N}$	vii.) The eq't $A\vec{x} = \vec{b}$ has at least one soln for each $\vec{b} \in \mathbb{R}^N$.
iii.) A has N pivot positions	viii.) The columns of A span $\mathbb{R}^N$.
iv.) $A\vec{x} = \vec{0}$ has only the trivial solution	ix.) (1 pt extra credit) the lin. trans $\vec{x} \mapsto A\vec{x}$ maps $\mathbb{R}^N$ onto $\mathbb{R}^N$
v.) the columns of A form a L.I. set.	x.) (1 pt extra credit) There is $C_{N \times N}$ s.t. $CA = I$

(xi)
There is $D_{N \times N}$ s.t. $AD = I$

(xii)
A^T is invertible

(xiii)
$\det(A) \neq 0$.