100	90's	80's	70's	60's	< 60
1	7	17	15	6	4

Assessment 1
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Math 220

high: 100
$\bar{x} = 78.1\%$

Name: key

I have found a very great number of exceedingly beautiful theorems.

Pierre de Fermat
1601 – 1665 (French mathematician)

No work = no credit med = 79.6%

No calculator

Warm-ups (1 pt each): $9+10=$ 19 $\qquad -\frac{0}{4}=$ 0 $\qquad -1^2=$ −1

R or "21"

1.) (1 pt) In the quote (above), Fermat refers to "beautiful theorems." What do you think makes a mathematical theorem beautiful? Answer using complete English sentences.

Simplicity, power, symmetry, and usefulness are qualities that can make theorems beautiful.

2.) (8 pts) The augmented matrix of a system is row reduced to $\left[\begin{array}{ccccc|c} 1 & -3 & 4 & -3 & 2 & 5 \\ 0 & 2 & -4 & 4 & 0 & -6 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$. $R_1 - 2R_3 \to R_1$, $\frac{1}{2}R_2 \to R_2$

Complete the row reduction process and write the solution to the system in vector form.

$$\sim \left[\begin{array}{cccccc} 1 & -3 & 4 & -3 & 0 & -3 \\ 0 & 1 & -2 & 2 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad R_1 + 3R_2 \mapsto R_1$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 0 & -12 \\ 0 & 1 & -2 & 2 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$x_1 = -12 + 2x_3 - 3x_4$
$x_2 = -3 + 2x_3 - 2x_4$
$x_3 = x_3$
$x_4 = x_4$
$x_5 = 4$

5/8 if rref correct but vecs in $\mathbb{R}^3$

$$\vec{x} = \begin{bmatrix} -12 \\ -3 \\ 0 \\ 0 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

3.) (8 pts) Consider the system $\begin{cases} 3x_1 + x_2 - 5x_3 = 9 \\ x_2 + 4x_3 = 0 \end{cases}$

a.) Write the system as a vector equation

$$x_1 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

b.) Write the system as a matrix equation

$$\begin{bmatrix} 3 & 1 & -5 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

3/4 if augmented

4.) (6 pts) True or False (circle one). Justify your answer.

a.) (T or F) The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.

False. Row ops reveal, but do not change the pivots.

b.) (T or F) The set Span$\{\vec{u},\vec{v}\}$ is always visualized as a plane through the origin.

False, It could also be a line or point.

c.) (T or F) If the equation $A\vec{x}=\vec{b}$ is inconsistent, then $\vec{b}$ is not in the set spanned by the columns of A.

True. $A\vec{x} = x_1\vec{a}_1 + \dots + x_n\vec{a}_n = \vec{b}$ where A has n cols. $A\vec{x}=\vec{b}$ being inconsistent means no lin. comb. of the cols of A (i.e. the span of A's cols) equals $\vec{b}$.

5.) (4 pts) Prove the following claim.

Claim: For all $\vec{u},\vec{v}\in\mathbb{R}^n$ and all scalars c: $c(\vec{u}+\vec{v})=c\vec{u}+c\vec{v}$

proof.

Let scalar c and vectors $\vec{u},\vec{v}\in\mathbb{R}^n$ be given.

$$c(\vec{u}+\vec{v}) = c\left(\begin{bmatrix}u_1\\ \vdots\\ u_n\end{bmatrix}+\begin{bmatrix}v_1\\ \vdots\\ v_n\end{bmatrix}\right) = c\begin{bmatrix}u_1+v_1\\ \vdots\\ u_n+v_n\end{bmatrix} = \begin{bmatrix}c(u_1+v_1)\\ \vdots\\ c(u_n+v_n)\end{bmatrix} = \begin{bmatrix}cu_1+cv_1\\ \vdots\\ cu_n+cv_n\end{bmatrix}$$

$$= \begin{bmatrix}cu_1\\ \\ cu_n\end{bmatrix}+\begin{bmatrix}cv_1\\ \\ cv_n\end{bmatrix} = c\begin{bmatrix}u_1\\ \vdots\\ u_n\end{bmatrix}+c\begin{bmatrix}v_1\\ \vdots\\ v_n\end{bmatrix} = c\vec{u}+c\vec{v}$$

Q.E.D.

6.) (8 pts) Solve the linear system using matrix methods

$$\begin{aligned} 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_1 \qquad - 3x_3 &= 8 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 9 & 7 \\ 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \end{array}\right] \begin{array}{l} R_1 \to R_3 \\ R_2 \to R \\ R_3 \to R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 2 & 2 & 9 & 7 \end{array}\right] R_3 - 2R_1 \to R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array}\right] R_3 - 2R_2 \to R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array}\right] \tfrac{1}{5}R_3 \mapsto R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{l} R_1 + 3R_3 \to R_1 \\ R_2 - 5R_3 \to R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

so $\vec{x} = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$