Math 220

1.8 Questions for flipped class

(1.8.1)

with T defined by $T(\mathbf{x}) = A\mathbf{x}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique.

$$A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

(1.8.2)

Let A be a 6×5 matrix. What must a and b be in order to define $T : \mathbb{R}^a \to \mathbb{R}^b$ by $T(\mathbf{x}) = A\mathbf{x}$?

(1.8.3)

use a rectangular coordinate system to plot $\mathbf{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and their images under the given transformation T. (Make a separate and reasonably large sketch for each exercise.) Describe geometrically what T does to each vector $\mathbf{x}$ in $\mathbb{R}^2$.

3. $T(\mathbf{x}) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

4. $T(\mathbf{x}) = \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

5. $T(\mathbf{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

6. $T(\mathbf{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(1.8.4)

19. Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbf{y}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $\mathbf{y}_2 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{e}_1$ into $\mathbf{y}_1$ and maps $\mathbf{e}_2$ into $\mathbf{y}_2$. Find the images of $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

(1.8.5)

1. (T/F) A linear transformation is a special type of function.

2. (T/F) Every matrix transformation is a linear transformation.

3. (T/F) If A is a 3×5 matrix and T is a transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, then the domain of T is $\mathbb{R}^3$.

4. (T/F) The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A.

5. (T/F) If A is an $m \times n$ matrix, then the range of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\mathbb{R}^m$.

6. (T/F) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and if $\mathbf{c}$ is in $\mathbb{R}^m$, then a uniqueness question is "Is $\mathbf{c}$ in the range of T?"

(1.8.6)

37. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = mx + b$.

a. Show that f is a linear transformation when $b = 0$.

b. Find a property of a linear transformation that is violated when $b \neq 0$.

c. Why is f called a linear function?

(1.8.1 solution)

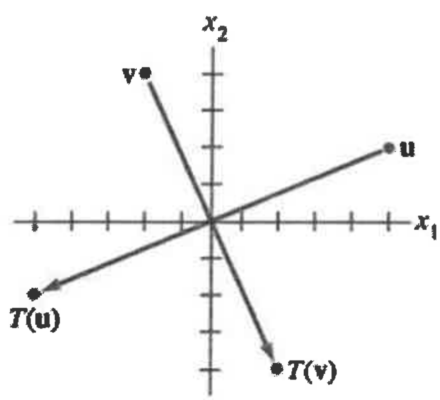
5. $\mathbf{x} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, not unique

(1.8.2 solution)

7. $a = 5, b = 6$

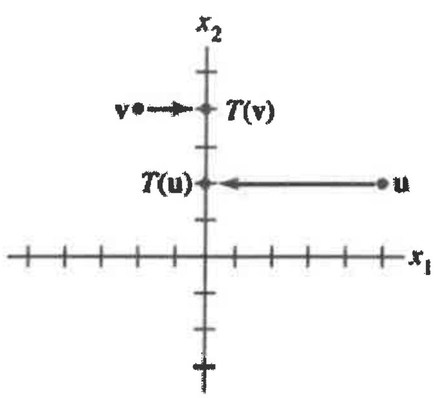
(1.8.3 solution)

13.



A reflection through the origin

15.



A projection onto the x_2-axis.

(1.8.4 solution)

19. $\begin{bmatrix} 13 \\ 7 \end{bmatrix}, \begin{bmatrix} 2x_1 - x_2 \\ 5x_1 + 6x_2 \end{bmatrix}$

(1.8.5 solution)

21. **(T/F)** A linear transformation is a special type of function. T

22. **(T/F)** Every matrix transformation is a linear transformation. T

23. **(T/F)** If A is a 3×5 matrix and T is a transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, then the domain of T is $\mathbb{R}^3$. F

24. **(T/F)** The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A. F

25. **(T/F)** If A is an $m \times n$ matrix, then the range of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\mathbb{R}^m$. F

26. **(T/F)** If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and if $\mathbf{c}$ is in $\mathbb{R}^m$, then a uniqueness question is "Is $\mathbf{c}$ in the range of T?" F

(1.8.6 solution)

37. **a.** When $b = 0$, $f(x) = mx$. In this case, for all x, y in $\mathbb{R}$ and all scalars c and d,

$$\begin{aligned} f(cx + dy) &= m(cx + dy) = mcx + mdy \\ &= c(mx) + d(my) = c \cdot f(x) + d \cdot f(y) \end{aligned}$$

This shows that f is linear.