Assessment 7
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Math 163

No work = no credit

Name: key

The divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever. By using them, one may draw any conclusion he pleases and that is why these series have produced so many fallacies and so many paradoxes ...

Niels Henrik Abel
1802-1829 (Norse mathematician)

Warm-ups (1 pt each): $\sum_{n=0}^{\infty} 0 = 0$ $\quad \sum_{n=0}^{\infty} 1 = \infty$ $\quad \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 2$

1.) (1 pt) We are just beginning to learn about power series (infinite series). Based upon the quote above, do you think Abel was trusted divergent series? Answer using complete English sentences.

Abel did not trust divergent series and thought them demonic.

2.) (5 pts) Write $\frac{3}{4+x}$ as a power series and state its interval of convergence.

$$\frac{3}{4+x} = \frac{3/4}{1-\left(-\frac{x}{4}\right)} = \sum_{n=0}^{\infty} \frac{3}{4}\left(-\frac{x}{4}\right)^n \text{ on } -4 < x < 4$$

3.) (5 pts) Consider the power series $f(x) = x^2 + \frac{x^3}{2} + \frac{x^4}{2^2} + \frac{x^5}{2^3} + \ldots$

a.) Write $f(x)$ using sigma notation

$$f(x) = \sum_{n=0}^{\infty} x^2 \cdot \left(\frac{x}{2}\right)^n$$

b.) Write a closed-form expression (not a power series) for $f(x)$

$$x^2 \cdot \frac{1}{1-\frac{x}{2}} = \frac{2x^2}{2-x}$$

4.) (5 pts) Find a power series representation for $g(x) = \dfrac{1}{(1+x)^2}$

$$\frac{d}{dx}\,\frac{-1}{1+x} = \frac{1}{(1+x)^2}$$

$$\Rightarrow \frac{1}{(1+x)^2} = \frac{d}{dx}\sum_{N=0}^{\infty} -1\cdot(-x)^N$$

$$= \sum_{N=0}^{\infty} -1\cdot(-1)^N \cdot N x^{N-1}$$

5.) (5 pts) Use the first three non-zero terms of a power series to approximate $\int_0^{0.2} \frac{1}{1-x^3}\,dx$.

Give your answer to five decimal places.

$$\frac{1}{1-x^3} = \sum_{N=0}^{\infty}(x^3)^N = 1 + x^3 + x^6 + \dots$$

$$\Rightarrow \int_0^{0.2}\frac{dx}{1-x^3} = \int_0^{0.2}(1 + x^3 + x^6)\,dx$$

$$= \left[x + \frac{x^4}{4} + \frac{x^7}{7}\right]_0^{0.2}$$

$$= 0.2 + \frac{0.2^4}{4} + \frac{0.2^7}{7}$$

$$\approx 0.20040$$