A6

Saturday, February 25, 2023 6:09 PM

Assessment 6
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Math 163

No work = no credit

Name: key

With the exception of the geometric series, there does not exist in all of mathematics a single infinite series whose sum has been determined rigorously.

Niels Henrik Abel
1802-1829 (Norse mathematician)

Warm-ups (1 pt each): $\frac{\partial}{\partial x}\arctan(x) = \frac{1}{1+x^2}$ $\frac{\partial}{\partial x}y^2 = 0$ $\frac{\partial}{\partial x}\ln(xy) = \frac{1}{x}$

1.) (1 pt) We are just beginning to learn about power series (infinite series). The example yesterday about investing related to the geometric series. Based upon the quote above, do you think Abel was satisfied with the way he had learned about infinite series? Answer using complete English sentences.

Abel was deeply dissatisfied w/ the lack of rigor around power series.

2.) (5 pts) If $z = 5x^2 + 3y$ and (x, y) changes from $(1,3)$ to $(1.05, 2.9)$ compare the values of Δz and dz.

$z_x = 10x\Big|_{(1,3)}$ 10

$z_y = 3$

so the tangent plane has equation

$z - 14 = 10(x-1) + 3(y-3).$

$\Rightarrow dz = 10(0.05) + 3(-0.1) = 0.2$

and

$\Delta z = \left[5(1.05)^2 + 3(2.9)\right] - 14 = 0.2125$

3.) (20 pts) Answer the following partial derivative questions.

a.) Consider $f(x,y) = 2y^5 \sin(xy^3)$

a. Find $f_x(x,y)$ $= 2y^8 \cdot \cos(xy^3).$

b. Find $\frac{\partial^2 f}{\partial y \partial x}$ $= 16y^7 \cos(xy^3) - 6y^{10} x \sin(xy^3)$

b.) Find the directional derivative of $g(p,w) = p^4 - p^2w^3$ at the point $(1,2)$ in the direction of the vector $\vec{v} = \langle 5,12 \rangle$.

$$\nabla g = \langle 4p^3 - 2pw^3, -3p^2w^2 \rangle \Big|_{(1,2)} \quad \langle -12, -12 \rangle$$

$$\vec{u} = \langle \tfrac{5}{13}, \tfrac{12}{13} \rangle$$

so $D_{\vec{u}} g = \langle -12, -12 \rangle \cdot \langle \frac{5}{13}, \frac{12}{13} \rangle = \frac{-204}{13}.$

c.) If $z = x^2y^3$, $x = \sin(st)$, and $y = s\ln(st)$, find $\frac{\partial z}{\partial s}$.

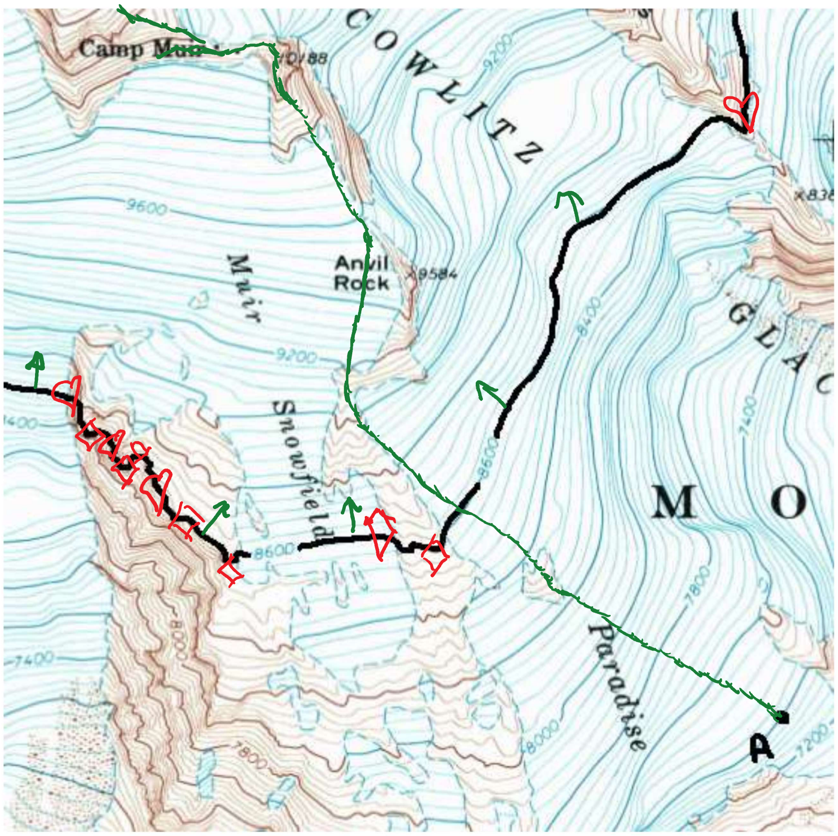
$$\begin{tikzcd}[column sep=small] & & z \arrow[dl, dash] \arrow[dr, dash] & & \\ & x \arrow[dl, dash] \arrow[d, dash] & & y \arrow[d, dash] \arrow[dr, dash] & \\ s & t & & s & t \end{tikzcd}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= 2xy^3 \cdot t\cos(st) + 3x^2y^2 \cdot \left(\ln st + s \cdot \tfrac{1}{st} \cdot t\right)$$

$$= 2xy^3 t \cos(st) + 3x^2y^2 (\ln(st)+1).$$

4.) (10 pts) Consider the contour plot (topographical map) of the glaciers near Paradise on Mt Rainier where $z = f(x,y)$ gives the altitude (in feet) at point (x,y) where x and y have the traditional orientation. The solid black line shows the level curve at 8,600 feet.



a.) What would happen if a hiker walked along the level curve $f(x,y) = 8{,}600$.

Their altitude would not change

b.) On the contour plot, clearly sketch at least 5 possible gradient vectors along the level curve $f(x,y) = 8{,}600$.

c.) On the contour plot, clearly mark with a diamond ◆ the point(s) of the level curve $f(x,y) = 8{,}600$ at which $f_x = 0$ and $f_y > 0$.

d.) On the contour plot, clearly mark with a heart ❤ the point(s) of the level curve $f(x,y) = 8{,}600$ at which the slope is greatest ($|\nabla f|$ is large).

e.) Beginning at point ***A***, clearly sketch the path of steepest ascent.