A3

Thursday, January 26, 2023 8:10 PM

Assessment 3
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Math 163

No work = no credit

Name: key

He is like the fox, who effaces his tracks in the sand with his tail.

Niels Henrik Abel
1802 - 1829 (Norwegian mathematician)

Warm-ups (1 pt each): $\vec{i} \times \vec{k} = -\vec{j}$ $\qquad \frac{2}{0} =$ undefined $\qquad -3^2 = -9$

1.) (1 pt) In the quote above, Abel talks about Gauss' writing style. According to Abel, how easy was it to understand Gauss' work? Answer using complete English sentences.

Gauss didn't show or explain his work.

2.) (5 pts) If $\vec{a} = \vec{i} + 3\vec{j} - 2\vec{k}$ and $\vec{b} = -\vec{i} + 5\vec{k}$ find $\vec{a} \times \vec{b}$

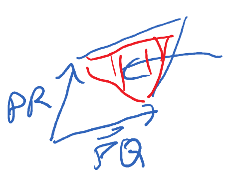
$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -2 \\ -1 & 0 & 5 \end{vmatrix} = \langle 15, -3, 3 \rangle$$

3.) (2 pts) Answer the following:

a.) Find a nonzero vector orthogonal to the plane through the points $P(1,0,1)$, $Q(-2,1,4)$, and $R(6,2,7)$.

$$\vec{PQ} = \langle -2, 1, 4 \rangle - \langle 1, 0, 1 \rangle = \langle -3, 1, 3 \rangle$$
$$\vec{PR} = \langle 6, 2, 7 \rangle - \langle 1, 0, 1 \rangle = \langle 5, 2, 6 \rangle$$
$$\Rightarrow \vec{PQ} \times \vec{PR} = \langle 0, -(-18-15), -11 \rangle = \langle 0, 33, 11 \rangle$$

b.) Find the area of the triangle PQR



Area of parallelogram $= |\langle 0, 33, 11 \rangle|$

Triangle Area $= \frac{1}{2}\sqrt{33^2 + 11^2} = \frac{11}{2}\sqrt{10}$.

4.) (4 pts) Find the symmetric AND vector equation(s) for the line of intersection of the planes $6x-3y-3z=0$ and $3x+y+z=5$.

Find 2 points on the line of intersection.

① Let $x=0$. $\Rightarrow \begin{cases} -3y-3z=0 \\ y+z=5 \end{cases} \Rightarrow \begin{matrix} y+z=20 \\ y+z=5 \end{matrix}$

No solution.

$\therefore x \neq 0$.

② Let $y=0$. $\Rightarrow \begin{cases} 6x-3z=0 \\ 3x+z=5 \end{cases} \Rightarrow -5z=-10$

$z=2$ AND $x=1$

③ Let $z=0$ $\Rightarrow \begin{cases} 6x-3y=0 \\ 3x+y=5 \end{cases} \Rightarrow y=2$ AND $x=1$

$P(1,0,2)$

and

$Q(1,2,0)$

$\Rightarrow \vec{PQ}=\langle 0,2,-2\rangle$

vector equation

$\vec{r}(t)=\langle 1,0,2\rangle+t\langle 0,2,-2\rangle$

symmetric equations

$1=\frac{y}{2}=\frac{z-2}{-2}$

5.) (4 pts) Find the equation of the plane that passes through $(6,0,-3)$ and contains the line $x=3-3t, y=2+5t, z=6+4t$.

Need 2 vectors on the plane.

① $\vec{u}=\langle -3,5,4\rangle$

② $\vec{v}=\langle 6,0,-3\rangle-\langle 3,2,6\rangle=\langle 3,-2,-9\rangle$

$\Rightarrow \vec{u}\times\vec{v}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 5 & 4 \\ 3 & -2 & -9 \end{vmatrix}=\langle -37,-15,-9\rangle$ normal vector

Plane: $-37(x-6)-15(y-0)-9(z+3)=0$.