

A2

Monday, January 23, 2023 8:38 PM

**Assessment 2**

Dusty Wilson

Math 163

**No work = no credit**Name: Keey

*It appears to me that if one wishes to make progress in mathematics, one should study the masters and not the pupils.*

Niels Henrik Abel

1802 - 1829 (Norwegian mathematician)

Warm-ups (1 pt each):

$9+10=19$

$-\frac{4}{0}=\text{undefined}$

$-1^2=-1$

1.) (1 pt) According to <sup>Abel</sup> ~~Ramsey~~ (above), who should we learn from? Answer using complete English sentences.

We do well to learn only from the best experts.

2.) (5 pts) If  $\vec{u} = \langle 0, -3, 4 \rangle$  and  $\vec{v} = \langle 2, -1, -2 \rangle$  find the following:

a.)  $|\vec{u}| = \sqrt{0^2 + (-3)^2 + 4^2} = \sqrt{25} = 5$

b.)  $5\vec{v} = 5\langle 2, -1, -2 \rangle = \langle 10, -5, -10 \rangle$

c.)  $\vec{u} + \vec{v} = \langle 0, -3, 4 \rangle + \langle 2, -1, -2 \rangle = \langle 2, -4, 2 \rangle$

d.)  $\vec{u} - \vec{v} = \langle 0, -3, 4 \rangle - \langle 2, -1, -2 \rangle = \langle -2, -2, 6 \rangle$

e.)  $3\vec{u} + 2\vec{v} = 3\langle 0, -3, 4 \rangle + 2\langle 2, -1, -2 \rangle = \langle 4, -11, 8 \rangle$

3.) (2 pts) Find  $\langle 1, -2, -1 \rangle \cdot \langle -6, 2, -3 \rangle$

$$= -6 + (-4) + 3$$

$$= -7.$$

Page 1 of 2

- 4.) (4 pts) Find a unit vector  $\bar{v}$  in the direction of the vector from  $P(1,0,1)$  to  $Q(3,2,0)$ .

$$\vec{PQ} = \langle 3, 2, 0 \rangle - \langle 1, 0, 1 \rangle = \langle 2, 2, -1 \rangle$$

$$|\vec{PQ}| = \sqrt{4 + 4 + 1} = 3$$

$$\text{unit vector} = \frac{1}{3} \langle 2, 2, -1 \rangle$$

- 5.) (4 pts) Find the angle between  $\bar{a} = \bar{i} - 2\bar{j} - 2\bar{k}$  and  $\bar{b} = 6\bar{i} + 3\bar{j} + 2\bar{k}$  (in radians).

$$\text{recall } \bar{a} \cdot \bar{b} = |\bar{a}| |\bar{b}| \cos \theta$$

$$\Rightarrow \theta = \cos^{-1} \left( \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| |\bar{b}|} \right)$$

$$= \cos^{-1} \left( -\frac{4}{21} \right)$$

$$\bar{a} \cdot \bar{b} = 6 + (-6) - 4$$

$$= -4$$

$$|\bar{a}| = \sqrt{1 + 4 + 4} = 3$$

$$|\bar{b}| = \sqrt{36 + 9 + 4} = 7$$

- 6.) (4 pts) Find the center and radius of the sphere  $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

complete the square.

$$\left( x^2 + 3x + \frac{9}{4} \right) + y^2 + \left( z^2 - 4z + 4 \right) = -1 + 4 + \frac{9}{4}$$

$$\left( \frac{3}{2} \right)^2 = \frac{9}{4} \Rightarrow \left( x + \frac{3}{2} \right)^2 + y^2 + (z - 2)^2 = \frac{21}{4}$$

$$\text{center @ } \left( -\frac{3}{2}, 0, 2 \right) \text{ w/ radius } \sqrt{\frac{21}{4}}$$