Power Series Math 163: Calculus III (Fall 2022)

Power Series, part 2

* Power Series term by term using derivatives

We have found the derivative of power series. This next example is a little different; here we use derivatives to find the coefficients.

Example 1: Find a power series representation for $f(x) = \sin(x)$.

Power Series Math 163: Calculus III (Fall 2022)

The series we found is an example of a more general type of power series called a Maclaurin Series.

<u>Definition</u>: Suppose the function f has derivatives of all orders on an interval centered at x = 0, then its

Maclaurin Series is: $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \dots$ This can be written more concisely as: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(4)}(0)}{n!}x^n$

Ch 11 Page 1

Power Series Math 163: Calculus III (Fall 2022)

The series we found is an example of a more general type of power series called a Maclaurin Series.

<u>Definition</u>: Suppose the function f has derivatives of all orders on an interval centered at x = 0, then its

Maclaurin Series is:
$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \dots$$

This can be written more concisely as: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

Note: A Maclaurin Series is a type of power series. It is found by finding the coefficients term by term using derivatives

Example 2: Find a Maclaurin Series (that is, a power series) representation for $f(x) = e^x$.

$$c_{0} = \frac{f(0)}{0!} = \frac{1}{1} = 1$$

$$c_{1} = \frac{f'(0)}{1!} = \frac{1}{1} = 1$$

$$c_{2} = \frac{f'(0)}{2!} = \frac{1}{2!}$$

$$c_{2} = \frac{f'(0)}{2!} = \frac{1}{2!}$$

$$c_{3} = \frac{f(3)(a)}{3!} = \frac{1}{3!}$$

$$c_{4} = \frac{f'(0)}{4!} = \frac{1}{4!}$$

$$c_{4} = \frac{1}{4!}$$

$$c_{4} = \frac{1}{4!}$$

Page **2** of **4**

As with geometric series, the Maclaurin Series can be manipulated to go quite a way:

Example 3: Find a Maclaurin Series (that is, a power series) representation for the following:

a.)
$$f(x) = e^{2x}$$

$$f(x) = e$$

If you have been paying attention, you may have noticed that we have found power series in the previous few examples, but have NOT made mention of where these series are valid. This is because we are no longer working with geometric series and consequently need more power (pun).

Specifically, we will use two facts and one method/test.

Fact 1: The Harmonic Series diverges. In symbols:
$$\sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{5} = \infty$$
Fact 2: The Alternating Harmonic Series converges. In symbols:
$$\sum_{n=1}^{\infty} \frac{1}{n} = \ln(2)$$

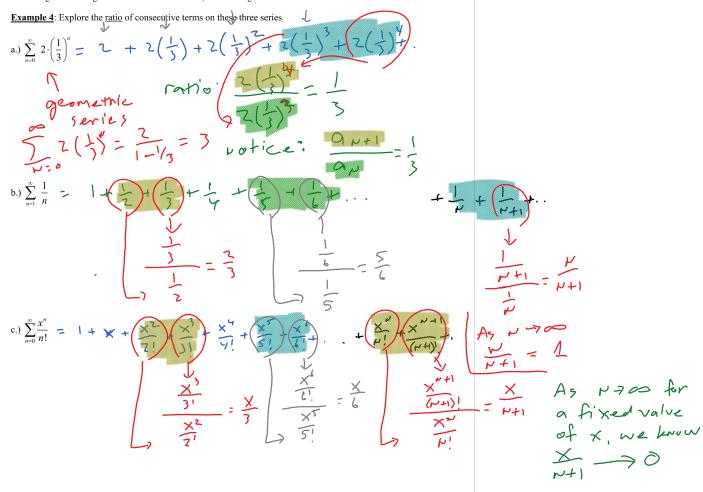
$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$$
Carverges

Page 3 of 4



So why does one infinite series converge and another diverge? That is a big question and we will leave it for another course. However, we can see that sometimes we sum an infinite number of terms and get a number (converge) and other times we don't get a number (diverge).

One of the most powerful ways of determining if a series will converge is to ask, "Do the terms decrease fast enough to converge? But how do we measure, "Fast enough"?



Page 4 of 4