A8

Thursday, December 8, 2022 12:41 PM

Assessment 8 **Name:** ______________________________
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Math 163

All truths are easy to understand once they are discovered; the point is to discover them.

Galileo Galilei
1564 - 1642 (Italian mathematician)

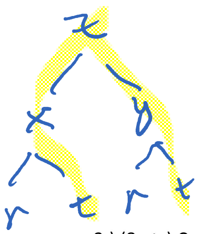
No work = no credit
No CAS Calculators

Warm-ups (1 pt each): $\frac{\partial}{\partial x}\frac{1}{x} = -\frac{1}{x^2}$ $\frac{\partial}{\partial x}\sqrt{x} = \frac{1}{2\sqrt{x}}$ $\frac{\partial}{\partial x}\tan^{-1}(x) = \frac{1}{1+x^2}$

1.) (1 pt) Please paraphrase the quote by Galileo (above). Answer using complete English sentences.

The challenge is in discovering what you don't already know.

2.) (8 pts) If $z = x^3y^9$, $x = r\cos(t)$, and $y = r\sin(t)$, find $\frac{\partial z}{\partial t}$.



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$= 3x^2y^9(-r\sin t) + 9x^3y^8(r\cos(t)).$$

3.) (8 pts) Consider the function $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ at $(2,1)$.

a.) Find the linearization and use it to approximate $f(1.95, 1.12)$.

$$\nabla f = \left\langle \frac{-x}{\sqrt{20-x^2-7y^2}}, \frac{-7y}{\sqrt{20-x^2-y^2}} \right\rangle \Big|_{(2,1)} \quad \left\langle \underset{f_x}{-\frac{2}{3}}, \underset{f_y}{-\frac{7}{3}} \right\rangle$$

Also: $f(2,1) = 3$. Linearization: $L(x,y) = -\frac{2}{3}(x-2) - \frac{7}{3}(y-1) + 3$.

b.) Find the directional derivative in the direction of the vector $\vec{v} = \langle -5, 12 \rangle$

$\hat{u} = \langle -\frac{5}{13}, \frac{12}{13} \rangle$

$D_{\hat{u}} f = \langle -\frac{2}{3}, -\frac{7}{3} \rangle \cdot \langle -\frac{5}{13}, \frac{12}{13} \rangle$

$= -\frac{74}{39}$

side check
$(2,1)$ to $(1.95, 1.12)$ is a distance of 0.13

$-\frac{74}{39}(0.13) = -0.246$

$f(1.95, 1.12) \approx L(1.95, 1.12)$

$= -\frac{2}{3}(-.05) - \frac{7}{3}(0.12) + 3$

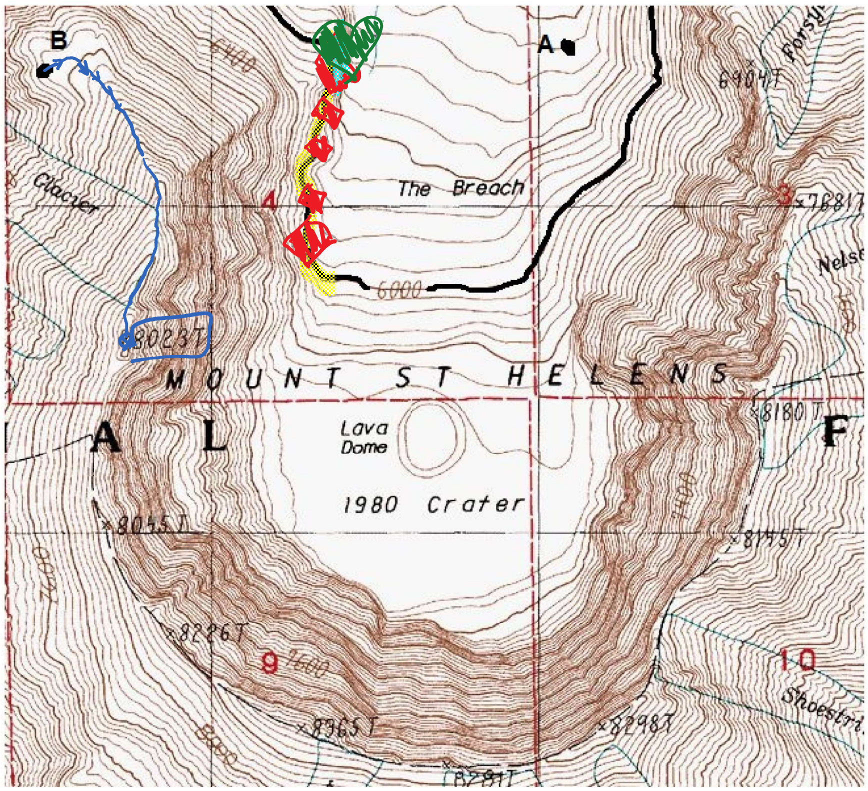
$= 2.753$

c.) If (x,y) changes from $(2,1)$ to $(1.95, 1.12)$, compare the exact change Δz and approximate change dz

$\Delta z \approx 2.7234 - 3 = -0.2766$ vs. the actual of 2.7234

$dz = -0.2467$

4.) (8 pts) Consider the contour plot (topographical map) of the crater of Mt Saint Helens where $z = f(x,y)$ gives the altitude (in feet) at point (x,y) where *x* and *y* have the traditional orientation. The solid black line shows the level curve at 6,000 feet.



a.) On the contour plot, clearly mark with a diamond ◆ the point(s) of the level curve $f(x,y) = 6{,}000$ at which $f_x < 0$ and $f_y = 0$.

b.) On the contour plot, clearly mark with a heart ♥ the point(s) of the level curve $f(x,y) = 6{,}000$ at which the slope is steepest ($|\nabla f|$ is large).

c.) Beginning at point ***B***, clearly sketch the path of steepest ascent.

5.) (0 pts). Would you like this Assessment to count double? **Clearly** circle one: YES NO.