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Assessment 6
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Math 163

Name: key

[Mathematics] is a kind of public language that allows us, as best we can, to try to achieve objectivity and certainty.

Javier Leach
1942 - 2016 (Spanish mathematician, theologian, priest)

No work = no credit
No CAS Calculators

Warm-ups (1 pt each): $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$ $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ [1]

1.) (1 pt) The quote by Leach (above) is from the author of a recent reading. What does Leach mean when he calls math a "public language?" Answer using complete English sentences.

understanding, using, and interpretating math is the same for all.

2.) (8 pts) Consider $f(x) = \frac{1}{x^2}$

a.) Find the Taylor Series at $x = 1$

$f(x) = \frac{1}{x^2} = x^{-2}$	1
$f'(x) = -2x^{-3}$	-2
$f''(x) = 6x^{-4}$	6
$f'''(x) = -24x^{-5}$	-24
$f^{(4)}(x) = 120x^{-6}$	120

$f(x) = \frac{1}{0!} - \frac{2}{1!}(x-1) + \frac{6(x-1)^2}{2!} - \frac{24}{3!}(x-1)^3 + \frac{120}{4!}(x-1)^4 - \dots$

$= 1 - 2(x-1) + 3(x-1)^2 - 4(x-1)^3 + 5(x-1)^4 - \dots$

$= \sum_{n=0}^{\infty} (-1)^n (n+1)(x-1)^n$

b.) Estimate $f(\frac{1}{2}) = 4$ using T_2.

$T_2(x) = 1 - 2(x-1) + 3(x-1)^2 \Rightarrow T_2(\frac{1}{2}) = 1 - 2(-\frac{1}{2}) + 3(-\frac{1}{2})^2$

$= 1 + 1 + \frac{3}{4}$

$= 2.75$

c.) What is the error in your estimate in (b.)?

error $4 - 2.75 = 1.25$

[1] What is the MacLaurin series for cosine?

3.) (8 pts) Find the MacLaurin series for $\sin(2x)$

	@ $x=0$
$\sin 2x$	0
$2\cos 2x$	2
$-4\sin 2x$	0
$-8\cos 2x$	-8
$+16\sin 2x$	0

$$\sin 2x = 0 + \frac{2x}{1!} + 0x^3 - \frac{8x^3}{3!} + 0x^4 + \frac{2^5}{5!}x^5 + 0x^6 - \frac{2^7}{7!}x^7 + \dots$$

$$= 2x - \frac{2^3}{3!}x^3 + \frac{2^5}{5!}x^5 - \frac{2^7}{7!}x^7 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{2n+1}\, x^{2n+1}}{(2n+1)!}$$

4.) (8 pts) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$

ratio test

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)(x+3)^{n+1}}{5^{n+1}}}{\frac{n(x+3)^n}{5^n}} \right| = \lim_{n\to\infty} \left| \frac{n+1}{n} \cdot \frac{(x+3)}{5} \right|$$

$\frac{n+1}{n} \to 1$

$$= \left| \frac{x+3}{5} \right| < 1$$

$\Rightarrow |x+3| < 5$

$\Rightarrow -5 < x+3 < 5$

$\Rightarrow -8 < x < 2$

R.O.C. $R = 5$