A5

Thursday, November 3, 2022 8:04 PM

Assessment 5
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Math 163

Name: key

8:08
8:18

The infinite! No other question has ever moved so profoundly the spirit of man.

David Hilbert
1862 - 1943 (Prussian mathematician)

No work = no credit
No CAS Calculators

Warm-ups (1 pt each): $\frac{3}{0} =$ undefined $\quad \vec{j} \cdot \vec{k} =$ 0 $\quad \vec{j} \times \vec{k} =$ $\vec{i}$

1.) (1 pt) According to Hilbert (above), what is the most profound question ever asked? Answer using complete English sentences.

Hilbert found the infinite profound.

2.) (8 pts) Answer the following

a.) What is the formula to find 7!

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

b.) Find $\frac{1000!}{999!}$ $= \frac{1000 \cdot \cancel{999 \cdot 998 \cdot \ldots \cdot 3 \cdot 2 \cdot 1}}{\cancel{999 \cdot 998 \cdot \ldots \cdot 3 \cdot 2 \cdot 1}} = 1000$

c.) Evaluate $\lim\limits_{k \to \infty} \frac{2\left[1 - \cancel{\left(\frac{1}{3}\right)^{k+1}}^{\to 0}\right]}{1 - \frac{1}{3}} = \frac{2}{2/3} = 3$

d.) $a + ax + ax^2 + ax^3 + ax^4 + \ldots$ is called a geometric series. It converges (equals a number) when:

$|x| < 1$.

3.) (8 pts) Find a power series representation for $\frac{3}{4+x}$. When does this series converge (equal a number)?

$$\frac{3}{4+x} = \frac{3}{4(1+\frac{x}{4})} = \frac{3/4}{1+\frac{x}{4}} = \frac{3/4}{1-(-\frac{x}{4})}$$

$$= \sum_{N=0}^{\infty} \frac{3}{4}\left(-\frac{x}{4}\right)^N = \sum_{N=0}^{\infty} \frac{3(-1)^N x^N}{4^{N+1}}$$

ok

converges when

$$\left|-\frac{x}{4}\right| < 1 \Rightarrow \left|\frac{x}{4}\right| < 1 \Rightarrow |x| < 4$$

4.) (8 pts) Integrate $\int \frac{1}{1-81x^4}dx$ using power series. When does this series converge (equal a number)?

$$\int \frac{1}{1-81x^4}dx = \int \sum_{N=0}^{\infty} 1(81x^4)^N \, dx = \int \sum_{N=0}^{\infty} 81^N x^{4N} \, dx$$

$$= \sum_{N=0}^{\infty} \int 81^N x^{4N} \, dx = C + \sum_{N=0}^{\infty} \frac{81^N x^{4N+1}}{4N+1}$$

This converges when

$$|81x^4| < 1 \Rightarrow |x^4| < \frac{1}{81} \Rightarrow |x| < \frac{1}{3}.$$