A4

Thursday, October 27, 2022 8:24 AM

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Assessment 4
Dusty Wilson
Math 163

Name: key

The enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious.

Eugene Wigner
1902 - 1995 (Hungarian Physicis)

No work = no credit
No CAS Calculators

Warm-ups (1 pt each): $\frac{3}{0} =$ undefined $\quad \vec{j}\cdot\vec{j} =$ 1 $\quad \vec{k}\times\vec{j} =$ $-\vec{i}$

1.) (1 pt) The quote by Wigner (above) is from the reading for this week. According to Wigner, how ought we to explain the usefulness of mathematics? Answer using complete English sentences.

Wigner thought the usefulness of math unexplainable.

2.) (12 pts) Answer the following

a.) A parametric representation for a circle with radius 2 that is centered at the origin is: $\vec{r}(t) =$ $\langle 2\cos t, 2\sin t\rangle$ on $0 \le t \le 2\pi$

b.) A parametric representation for the function $y = x^2$ is: $\vec{r}(t) =$ $\langle t, t^2\rangle$

c.) I bought a "new" car this week (true story). As I was testing it out, I tried to see how quickly it could get up to speed and then also tested the brakes coming down a steep hill. In the vocabulary of this class, what was I exploring?

the tangential component of acceleration (a_T)

d.) If you are given an acceleration vector-valued function $\vec{a}(t)$, how would you go about finding the position function? What (if any) additional information would you need?

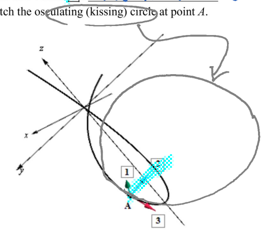
$\vec{v}(t) = \int \vec{a}(t)\,dt$

$\vec{r}(t) = \int \vec{v}(t)\,dt$

1. integrate
2. need 2 pieces of info about $\vec{v}$ and $\vec{r}$ to find the constants of integration.

e.) Consider the graph below. What is the name of the unit vector 2: unit normal vector

Sketch the osculating (kissing) circle at point A.



3.) (16 pts) Consider the space curve $\vec{r}(t) = \langle 5\cos t, 5\sin t, 12t\rangle$

a.) Find the curvature of $\vec{r}(t)$

1. $\vec{r}\,' = \langle -5\sin t, 5\cos t, 12\rangle$
2. $\vec{r}\,'' = \langle -5\cos t, -5\sin t, 0\rangle$
3. $\vec{r}\,'\times\vec{r}\,'' = \langle 60\sin t, -60\cos t, 25\sin^2 t + 25\cos^2 t\rangle$
   $= \langle 60\sin t, -60\cos t, 25\rangle$
4. $|\vec{r}\,'| = \sqrt{25\sin^2 t + 25\cos^2 t + 144} = 13$
5. $|\vec{r}\,'\times\vec{r}\,''| = \sqrt{60^2\sin^2 t + 60^2\cos^2 t + 625}$
   $= \sqrt{3600 + 625} = 65$

Thus: $k = 65/13^3 = 5/169$.

b.) At the point $A(-5, 0, 12\pi)$, the normal vector to $\vec{r}(t)$ is $\vec{N} = \langle 1, 0, 0\rangle$. Find the equation of the osculating plane to $\vec{r}(t)$ at point A.

point $(-5, 0, 12\pi)$
orthogonal vec. binormal: $\vec{B}$.

1. $\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \left\langle -\frac{5}{13}\sin t, \frac{5}{13}\cos t, \frac{12}{13}\right\rangle\Big|_{t=\pi}$ $\left\langle 0, -\frac{5}{13}, \frac{12}{13}\right\rangle$
2. Find $\vec{N}(\pi)$
   1st: $\vec{T}\,' = \left\langle -\frac{5}{13}\cos t, -\frac{5}{13}\sin t, 0\right\rangle$
   2nd: $|\vec{T}\,'| = \frac{5}{13}$
   3rd: $\vec{N}(t) = \langle -\cos t, -\sin t, 0\rangle\Big|_{t=\pi}$ $\langle 1, 0, 0\rangle$
3. Find $\vec{B} = \vec{T}\times\vec{N}$

$\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & \ldots & 12\ldots \end{vmatrix} = \left\langle 0, \frac{12}{\ldots}, \frac{5}{\ldots}\right\rangle$

③ Find $\vec{B} = \vec{T} \times \vec{N}$

$$\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -5/13 & 12/13 \\ 1 & 0 & 0 \end{vmatrix} = \langle 0, \frac{12}{13}, \frac{5}{13} \rangle$$

④ Plane: $0(x+5) + \frac{12}{13}(y-0) + \frac{5}{13}(z-12\pi) = 0$