A3

Thursday, October 13, 2022 8:29 AM

Assessment3.22.4

Assessment 3
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Math 254

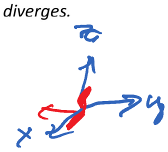
Name: Key

8:34
8:51

Whenever an infinite series is obtained as the development of some closed expression [formula for a function], *it may be used in mathematical operations as the equivalent of that expression, even for values of the variable for which the series diverges.*

Leonhard Euler
1707 – 1783 (Swiss Mathematician)

No work = no credit
No CAS Calculators



Warm-ups (1 pt each): $-3^2 =$ -9 $\quad \vec{i}\cdot\vec{j} =$ 0 $\quad \vec{i}\times\vec{k} =$ $-\vec{j}$

1.) (1 pt) The quote by Euler (above) is from our ~~reading for this week~~. According to Euler, when was it acceptable to use divergent series? Answer using complete English sentences.

We may use divergent series provided they were found during the development of some closed expression.

2.) (8 pt) Consider the points $A(1,2,3)$, $B(6,5,4)$, and $C(9,8,7)$.

a.) Find symmetric equations for the line that includes points A and B.

$\vec{AB} = \langle 5, 3, 1\rangle$

line: $\vec{r}(t) = \langle 1,2,3\rangle + t\langle 5,3,1\rangle$

$x = 1 + 5t\ ;\ y = 2 + 3t\ ;\ z = 3 + t$

$\dfrac{x-1}{5} = \dfrac{y-2}{3} = z-3$

b.) Find the equation of the plane that includes points A, B, and C.

$\vec{AB} = \langle 5, 3, 1\rangle$

$\vec{AC} = \langle 8, 6, 4\rangle$

$\vec{AB}\times\vec{AC} = \langle 6, -12, 6\rangle$

plane: $6(x-1) - 12(y-2) + 6(z-3) = 0$

$6x - 12y + 6z = 6 - 24 + 18 = 0$

3.) (8 pt) Complete the following

a.) Find the parametric equations for the line through the point $A(1,2,3)$ and perpendicular to both $\langle 5,3,1\rangle$ and $8\vec{i}+6\vec{j}+4\vec{k}$. $\longleftarrow$ cross-product $\langle 6,-12,6\rangle$

line: $\vec{r}(t) = \langle 1,2,3\rangle + t\langle 6,-12,6\rangle$

$(x(t), y(t), z(t)) = (1+6t,\ 2-12t,\ 3+6t)$

b.) Find the equation of the plane that passes through the point $A(1,2,3)$ and contains the line of intersection of the planes $5x+3y+z=5$ and $8x+6y+4z=14$.

① direction of line.

$\langle 5,3,1\rangle \times \langle 8,6,4\rangle = \langle 6,-12,6\rangle$

(vector on/parallel to plane)

② point on line.

let $x=0 \Rightarrow \begin{cases} 3y + z = 5 \\ 6y + 4z = 14 \end{cases} \Rightarrow 2z = 4 \Rightarrow z = 2$ and $y=1$. point $(0,1,2)$

③ find a 2nd vec: $\langle 1,2,3\rangle - \langle 0,1,2\rangle = \langle 1,1,1\rangle$

④ cross-product

$\langle 6,-12,6\rangle \times \langle 1,1,1\rangle$

$\vec{n} = \langle -18, -0, 18\rangle$

plane

$-18(x-1)+0(y-2)+18(z-3)=0$

$-18x+18z = -18+54 = 36$

4.) (4 pt) Consider the parametric equations $x=t^2$ and $y=t^4$.

a.) Eliminate the parameter to find a Cartesian equation of the curve.

$y = (t^2)^2$

$\Rightarrow y = x^2,\ x \geq 0$

b.) Keeping in mind the original parametric equations, what is the domain of your Cartesian equation?

Domain: $[0,\infty)$.