4: 20

Assessment 2
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Math 163

Name: Key

... arithmetic has a very great and elevating effect, compelling the soul to reason about abstract number, and rebelling against the introduction of visible or tangible objects into the argument.

Plato
circa 428 – 348 BC (Greek Philosopher)

No work = no credit
No CAS Calculators

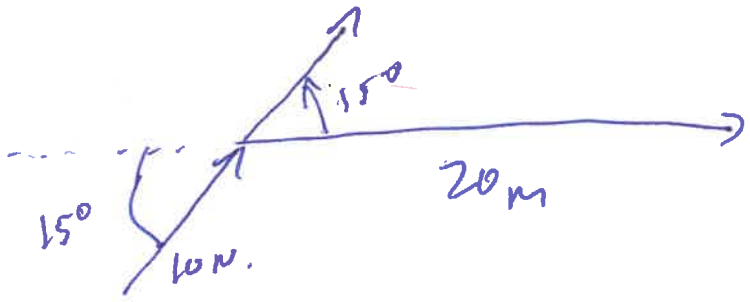
Warm-ups (1 pt each): $-3^2 = -9$ $\vec{i} \cdot \vec{j} = 0$ $\vec{i} \times \vec{k} = -\vec{j}$

1.) (1 pt) The quote by Plato (above) is from the same book as the Allegory of the Cave. According to Plato, what is it about numbers that distinguishes them from visible or tangible objects? Answer using complete English sentences.

Numbers elevate the mind thru abstraction.

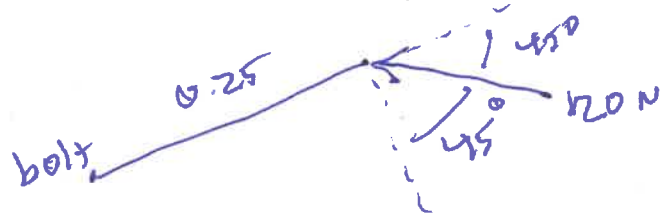
2.) (4 pts) Applications

a.) How much work is done when a stroller is pushed 20 m along a horizontal sidewalk with a constant force of 10 N at an angle of 15° below the horizontal?



$W = \vec{F} \cdot \vec{D}$
$= 10(20)\cos(15°)$
$= 193.2$ N-m

b.) Suppose you apply a force of 20 N to a 0.25-meter-long wrench attached to a bolt in a direction perpendicular to the bolt. Determine the magnitude of the torque when the force is applied at an angle of 45° to the wrench.



$\tau = |\vec{F} \times \vec{D}|$
$= |\vec{F}||\vec{D}| \sin 45°$
$= 20(0.25)\frac{\sqrt{2}}{2}$
$= \frac{5\sqrt{2}}{2}$ N-m

3.) (8 pts) Consider $\vec{a} = \langle -8, 0, 2 \rangle$ and $\vec{b} = \langle 1, 3, -3 \rangle$, find the following.

a.) $\vec{a} \cdot \vec{b} = \langle -8, 0, 2 \rangle \cdot \langle 1, 3, -3 \rangle = -8 + 0 - 6 = -14$

b.) The angle between vectors $\vec{a}$ and $\vec{b}$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right) = \cos^{-1}\left(\frac{-14}{\sqrt{68}\sqrt{19}}\right) = 112.9^\circ$$

OR

1.97 rad

c.) $|\vec{a}|^2 = \vec{a} \cdot \vec{a} = 68$

d.) $\text{proj}_{\vec{a}}\vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\right)\vec{a} = \frac{-14}{68}\langle -8, 0, 2 \rangle$

4.) (8 pts) Consider $\vec{a} = \langle -8, 0, 2 \rangle$ and $\vec{b} = \langle 1, 3, -3 \rangle$, find the following.

a.) $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -8 & 0 & 2 \\ 1 & 3 & -3 \end{vmatrix} = \langle -6, -22, -24 \rangle$

b.) Find the area of the parallelogram that has the two adjacent sides $\vec{a}$ and $\vec{b}$

$$\text{Area} = |\vec{a} \times \vec{b}| = \sqrt{6^2 + 22^2 + 24^2} = \sqrt{1096}$$

$$\approx 33.1$$