

Name: _____

Assessment 6

Math 220: Linear Algebra

Instructions: Please carefully complete these questions by hand. Be sure to show all work (this includes notating steps in row reduction in matrices that include a variable).

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable. Write your name on every page. You can submit more pages, but Gradescope will not accept less pages than the original assignment.

Upload your solutions to Gradescope by 8 am on Monday (3/8). During your presentation time, you will be asked to explain your thought process and reasoning on a randomly assigned question. Late submissions (or resubmissions) are available thru 5 pm with a 5% penalty. Resubmission is helpful if you think you can gain 5% in the process.

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

<https://docs.google.com/spreadsheets/d/1TE17S-z6oWrdejMbX5txwWX-JowaTFspdJ-7pCtpodY/edit?usp=sharing>

Reminders: It is okay to collaborate with peers and use online resources. However, the final work should be your own and you should be prepared to present on each question.

(1.1) If the nullity of a 4×6 matrix A is 3, what are the dimensions of the column and row spaces of A ?

(1.2) Consider $A = \begin{bmatrix} 1 & 0 & 7 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Determine a basis for, and find the dimensions of Nul A, Col A, and Row A.

(1.3) If $\vec{v}_1, \dots, \vec{v}_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \dots, \lambda_r$ of an $n \times n$ matrix A , prove that the set $\{\vec{v}_1, \dots, \vec{v}_r\}$ is linearly independent.

(1.4) In P_2 , find the change-of-coordinates matrix from $B = \{1 + 2t^2, 3 + t + 7t^2, 1 - 3t\}$ to the standard basis. Then write t^2 as a linear combination of the polynomials in B. Please clearly describe any mappings to/from P_2 .

(1.5) Let $B = \left\{ \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \end{bmatrix} \right\}$ and $C = \left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ be bases for \mathbb{R}^2 . Find the change-of-coordinates matrix from C to B. Draw the diagram that relates $\vec{x}, [\vec{x}]_B, [\vec{x}]_C$.

(1.6) Let λ be an eigenvalue of an invertible matrix A . Prove that λ^{-1} is an eigenvalue of A^{-1} .

(1.7) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 0 & 0 \\ -8 & 0 & -4 \end{bmatrix}$.

(1.8) Find the eigenspace of $A = \begin{bmatrix} 6 & 0 & 1 & 0 \\ 3 & 4 & 1 & 0 \\ 4 & -3 & 3 & 0 \\ 2 & -4 & -6 & 5 \end{bmatrix}$ corresponding to the eigenvalue $\lambda = 5$.