Name: ________________________
Assessment 5
Math 220: Linear Algebra

Instructions: Please carefully complete these questions by hand. Be sure to show all work (this includes notating steps in row reduction in matrices that include a variable).

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable. Write your name on every page. You can submit more pages, but Gradescope will not accept less pages than the original assignment.

Upload your solutions to Gradescope by 8 am on Monday (3/1). During your presentation time, you will be asked to explain your thought process and reasoning on a randomly assigned question. Late submissions (or resubmissions) are available thru 5 pm with a 5% penalty. Resubmission is helpful if you think you can gain 5% in the process.

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

https://docs.google.com/spreadsheets/d/1TE17S-z6oWrdejMbX5txwWX-JowaTFspdJ-7pCtpodY/edit?usp=sharing

Reminders: It is okay to collaborate with peers and use online resources. However, the final work should be your own and you should be prepared to present on each question.

(1.1) Suppose $\mathbb{R}^4 = \text{Span}\{\vec{v}_1, \ldots \vec{v}_4\}$. Explain why $\{\vec{v}_1, \ldots \vec{v}_4\}$ is a basis for $\mathbb{R}^4$.

(1.2) Find a basis for the space spanned by $\begin{bmatrix} 4 \\ 9 \\ -5 \\ -6 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 4 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ -7 \\ 1 \\ -7 \end{bmatrix}, \begin{bmatrix} 3 \\ -15 \\ 27 \\ 9 \\ 12 \end{bmatrix}, \begin{bmatrix} -30 \\ -41 \\ -7 \\ 26 \\ -22 \end{bmatrix}$. Justify your work.

(1.3) Suppose that $\{\vec{v}_1, \ldots \vec{v}_p\}$ is a subset of V and T is a one-to-one linear transformation, so that an equation $T(\vec{u}) = T(\vec{v})$ always implies that $\vec{u} = \vec{v}$. Prove that if the set of images $\{T(\vec{v}_1), \ldots T(\vec{v}_p)\}$ is linearly dependent, then $\{\vec{v}_1, \ldots \vec{v}_p\}$ is linearly dependent.

(1.4) State the Invertible Matrix Theorem as give through section 4.5 (this requires listing assumptions and then 17+1 equivalent statements).

(1.5) Let $\vec{v}_1 = \begin{bmatrix} 7 \\ -6 \\ 4 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 9 \\ -5 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 18 \\ 1 \\ 1 \end{bmatrix}$, and $H = \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. Verify that the vectors are linearly dependent and use this information to find a basis for *H*.

(1.6) Let $H = \text{Span}\left\{ \overset{\vec{v}_1}{\begin{bmatrix} 12 \\ -6 \\ 11 \\ 7 \end{bmatrix}}, \overset{\vec{v}_2}{\begin{bmatrix} 15 \\ -9 \\ 14 \\ 10 \end{bmatrix}} \right\}$ and $B = \{\vec{v}_1, \vec{v}_2\}$. Show that $\vec{x} = \begin{bmatrix} 17 \\ -11 \\ 16 \\ 12 \end{bmatrix}$ is in *H* and find the *B*-coordinate of $\vec{x}$.

(1.7) Determine whether the vectors $p_1(t) = 3 + 7t$, $p_2(t) = 4 + 2t - 3t^3$, $p_3(t) = 4t - 2t^2$, and $p_4(t) = 2 + 28t - 8t^2 + 3t^3$ form a basis for P_3. Justify your conclusions.

(1.8) State and prove the spanning set theorem.

(1.9) If $B = \left\{ \begin{bmatrix} 8 \\ -7 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}$, use two different methods to find the B-coordinates of $\vec{x} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

(1.10) Find the change-of-coordinates matrix from the standard basis in $\mathbb{R}^3$ to B-coordinates where $B = \left\{ \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -5 \\ 7 \end{bmatrix} \right\}$. Give your answer without decimals and with fully reduced fractions.