Name: ________________________
Assessment 4
Math 220: Linear Algebra

Instructions: Please carefully complete these questions by hand. Be sure to show all work (this includes notating steps in row reduction in matrices that include a variable).

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable. Write your name on every page. You can submit more pages, but Gradescope will not accept less pages than the original assignment.

Upload your solutions to Gradescope by 8 am on Monday (2/22). During your presentation time, you will be asked to explain your thought process and reasoning on a randomly assigned question. Late submissions (or resubmissions) are available thru 5 pm with a 5% penalty. Resubmission is helpful if you think you can gain 5% in the process.

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

https://docs.google.com/spreadsheets/d/1TE17S-z6oWrdejMbX5txwWX-JowaTFspdJ-7pCtpodY/edit?usp=sharing

Reminders: It is okay to collaborate with peers and use online resources. However, the final work should be your own and you should be prepared to present on each question.

(1.1) Let $A=\begin{bmatrix} -2 & -4 & -16 \\ 3 & 5 & 19 \\ 1 & 3 & 14 \end{bmatrix}$. Find the third column of A^{-1} without computing the other two columns.

Give exact answers. Explain how this works.

(1.2) Find the inverse of $A = \begin{bmatrix} 1 & -2 & 1 \\ -5 & 11 & -6 \\ -4 & -1 & 5 \end{bmatrix}$, if it exists. Use the algorithm for finding A^{-1} that involves row reducing $\left[A \mid I \right]$. <u>If *A* is invertible, explain why this means the columns of *A* are linearly independent</u>.

(1.3) Let *A* and *B* be $n \times n$ matrices. Prove that if *AB* is invertible, so is *B*.

Use the proof structure taught in class, in the videos, and in the class notes.

(1.4) Consider the linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ where $T\left(x_1, x_2\right) = \left(2x_1 - 7x_2, -2x_1 + 6x_2\right)$. Show that T is invertible and find a formula for T^{-1}. Your result should be in the same notation as the T provided.

(1.5) By hand, compute $\begin{vmatrix} -1 & 3 & 6 & 0 \\ 4 & 5 & 3 & 0 \\ 4 & 4 & 6 & 8 \\ 4 & 2 & 4 & 4 \end{vmatrix}$. Is the associated matrix invertible? Why or why not.

(Checking with a calculator is recommended).

(1.6) By hand, compute $\det\left(B^3\right)$ where $B = \begin{bmatrix} 2 & 0 & 2 \\ 1 & 2 & 4 \\ 2 & 2 & 2 \end{bmatrix}$. (Checking with a calculator is recommended).

(1.7) Let H and K be subspaces of a vector space V. The intersection if H and K, written as $H \cap K$, is the set of $\vec{v} \in V$ that belong to both H and K. Prove that $H \cap K$ is a subspace of V.

Use the proof structure taught in class, in the videos, and in the class notes.

Give an example in $\mathbb{R}^2$ to show that the union of two subspaces is not, in general, a subspace.

(1.8) The set of $M_{2\times 2}$ of all 2×2 matrices is a vector space, under the usual operations of addition of matrices and multiplication by real scalars. Prove that the set H of all matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ is (<u>or isn't</u>) a subspace of $M_{2\times 2}$.

<u>Use the proof structure taught in class, in the videos, and in the class notes</u>.

(1.9) Determine whether the vector $\begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$ is in the column space of $A = \begin{bmatrix} -11 & 7 & 1 & 0 \\ -5 & 2 & 4 & 5 \\ 10 & -8 & 4 & 7 \\ 3 & -2 & 0 & 0 \end{bmatrix}$, the null space of *A*, both, or neither. Explain/justify your reasoning.

(1.10) Let $T: V \rightarrow W$ be a linear transformation from a vector space V into a vector space W. Prove that the range of T is a subspace of W.

Use the proof structure taught in class, in the videos, and in the class notes.