Name: _____
Assessment 3
Math 220: Linear Algebra

Instructions: Please carefully complete these questions by hand. Be sure to show all work (this includes notating steps in row reduction in matrices that include a variable).

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable. Write your name on every page. You can submit more pages, but Gradescope will not accept less pages than the original assignment.

Upload your solutions to Gradescope by 8 am on Monday (2/8). During your presentation time, you will be asked to explain your thought process and reasoning on a randomly assigned question. Late submissions (or resubmissions) are available thru 5 pm with a 5% penalty. Resubmission is helpful if you think you can gain 5% in the process.

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

https://docs.google.com/spreadsheets/d/1TE17S-z6oWrdejMbX5txwWX-JowaTFspdJ-7pCtpodY/edit?usp=sharing

Reminders: It is okay to collaborate with peers and use online resources. However, the final work should be your own and you should be prepared to present on each question.

(1.1) Let $\vec{b} = \begin{bmatrix} 5 \\ 19 \\ 41 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 4 & -2 & 5 & -9 \\ -5 & 7 & -8 & 0 \\ -8 & 10 & 5 & 3 \\ 6 & -4 & 8 & -11 \end{bmatrix}$. Is $\vec{b}$ in the range of the transformation $\vec{x} \to A\vec{x}$. If so, find all $\vec{x}$ whose image under the transformation is $\vec{b}$. Justify your answer.

(1.2) Let $T:\mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Prove that the set $\{T(\vec{v}_1), T(\vec{v}_2), T(\vec{v}_3)\}$ is linearly dependent.

Use the proof structure taught in class, in the videos, and in the class notes.

(1.3) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\vec{e}_1 \to \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\vec{e}_2 \to \begin{bmatrix} -2 \\ 8 \end{bmatrix}$. Find the images of $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and find the matrix of the linear transformation.

(1.4) Let T be the linear transformation matrix with standard matrix $A = \begin{bmatrix} 4 & -7 & 3 & -1 & 5 \\ 6 & -8 & 5 & 0 & -8 \\ -7 & 10 & -8 & 5 & 14 \\ 3 & -5 & 4 & -4 & -6 \\ -5 & 6 & -6 & 3 & 3 \end{bmatrix}$.

Is the transformation one-to-one? Is it onto? Justify your answers.

(1.5) Describe the possible echelon forms of the standard matrix for a linear transformation T where $T:\mathbb{R}^3 \to \mathbb{R}^5$ that would make the transformation (a.) one-to-one and (b.) onto. Please make sure to clearly define any special characters that you use.

(1.6) Determine if the linear transformation $T\left(x_1, x_2 x_3\right) = \left(x_1 - 5x_2 + x_3, x_2 - 7x_3\right)$ is <u>one-to-one</u> and <u>onto</u>. Justify your answer.

(1.7) If $\vec{u} = \begin{bmatrix} -9 \\ 2 \\ -6 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, compute $\vec{u}^T\vec{v}$, $\vec{v}^T\vec{u}$, $\vec{u}\vec{v}^T$, $\vec{v}\vec{u}^T$.

(1.8) If $A = \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 \\ -2 & k \end{bmatrix}$, what value(s) of *k*, if any, will make $AB = BA$.

(1.9) Prove that if $T:\mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then $T(\vec{x}) = \vec{0}$ has only the trivial solution if and only if T is one-to-one.

Use the proof structure taught in class, in the videos, and in the class notes.

(1.10) Explain how you would show a transformation is a linear transformation. Provide a mathematical example of a non-linear transformation (show that it does not meet the necessary conditions).