Name: _____

Assessment 2

Math 220: Linear Algebra

<u>Instructions</u>: Please carefully complete these questions by hand. Be sure to show all work (this includes notating steps in row reduction in matrices that include a variable).

Should you choose to work these on scratch paper, <mark>please do not put more than one question on a page</mark>. Additional sheets of paper are acceptable. Write your name on every page. <mark>You can submit more pages, but Gradescope will not accept less pages than the original assignment.</mark>

Upload your solutions to Gradescope by 8 am on Monday (2/1). During your presentation time, you will be asked to explain your thought process and reasoning on a randomly assigned question. Late submissions (or resubmissions) are available thru 5 pm with a 5% penalty. Resubmission is helpful if you think you can gain 5% in the process.

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

https://docs.google.com/spreadsheets/d/1TE17S-z6oWrdejMbX5txwWX-JowaTFspdJ-7pCtpodY/edit?usp=sharing

Reminders: It is okay to collaborate with peers and use online resources. However, the final work should be your own and you should be prepared to present on each question.

(1.1) Explain the significance/interpretation and prove that if $A\vec{x} = \vec{b}$ is consistent for some given $\vec{b}$ and $\vec{p}$ is a solution, then the solution set of $A\vec{x} = \vec{b}$ is the set of all vectors of the form $\vec{w} = \vec{p} + \vec{v}_h$, where $\vec{v}_h$ is any solution of the homogeneous equation $A\vec{x} = \vec{0}$.

Use the proof structure taught in class, in the videos, and in the class notes.

(1.2) Describe the solutions of the first system in parametric vector form. Provide a geometric comparison with the solution set of the second system of equations below.

$$
\begin{array}{rrrrrrl}
3x_1 & + & 3x_2 & + & 6x_3 & = & 12 \\
-9x_1 & - & 9x_2 & - & 18x_3 & = & -36 \\
 & & -4x_2 & - & 4x_3 & = & 16
\end{array}
\qquad \text{and} \qquad
\begin{array}{rrrrrrl}
3x_1 & + & 3x_2 & + & 6x_3 & = & 0 \\
-9x_1 & - & 9x_2 & - & 18x_3 & = & 0 \\
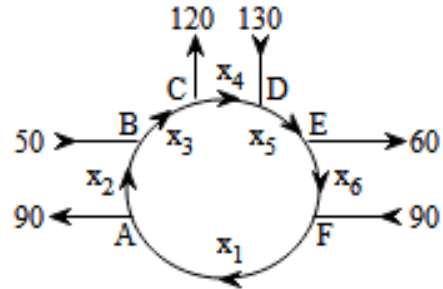 & & -4x_2 & - & 4x_3 & = & 0
\end{array}
$$

(1.3) If $\vec{b} \neq \vec{0}$, can the solution set of $A\vec{x} = \vec{b}$ be a plane through the origin? If yes, provide an example. If not, explain why not.

(1.4) Balance the chemical equation $C_2H_3Cl + O_2 \rightarrow CO_2 + H_2O + HCl$. What mathematical restrictions are there on the coefficient of HCl? What physics/chemical/atomic restrictions are there on the coefficient of HCl?

(1.5) In 2020, the City of Kent agreed to pay Active Construction, Inc. of Tacoma \$4.75 million to put in a roundabout at the intersection of 4th and Willis.[1] With that much money on the line, it made sense to do a little research. Car counting revealed the following traffic pattern during the 10am hour.

What are the max/min values for x_1 .

Should the contractors use a road that has the same number of lanes all the way around? Why or why not.

[1] https://www.kentreporter.com/news/kent-city-council-awards-4-7-million-fourth-and-willis-roundabout-project/

(1.6) Find the value(s) of h for which the vectors $\begin{bmatrix} 1 \\ -3 \\ -6 \end{bmatrix}, \begin{bmatrix} -3 \\ 10 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}$ are linearly dependent. Justify your

answer.

(1.7) Consider $A = \begin{bmatrix} 3 & -3 & -3 & 3 & 2 \\ 9 & -7 & 3 & 15 & 9 \\ 6 & 0 & 30 & 24 & 16 \\ 9 & -3 & 27 & 27 & 24 \end{bmatrix}$.

Use as many columns of A as possible to construct a matrix B with the property that the equation $B\vec{x} = \vec{0}$ has only the trivial solution. What can be said about any unused columns of A?

(1.8) Consider the set spanned by the columns of $B = \begin{bmatrix} 3 & -3 & 2 \\ 6 & -3 & 7 \\ 6 & 3 & 16 \\ 9 & -3 & 21 \end{bmatrix}$.

Please write an equation describing all vectors in the span of the columns of B. Can you write another equation describing all the vectors in $\mathbb{R}^4$ that are not in the span of the columns of B?

(1.9) Suppose A is a 4x3 matrix whose first two columns are linearly independent and whose third column isn't in the span of the preceding columns. Describe all possible reduced row echelon forms of A using the characters, 0, 1, and "*" where "*" represents entries that may have any value including 0 and 1.

(1.10) Prove that an indexed set $S = \{\vec{v}_1, ..., \vec{v}_p\}$ of two or more vectors is linearly dependent if and only if at least one of the vectors in S is a linear combination of the others. In fact, if S is linearly dependent and $\vec{v}_1 \neq \vec{0}$, then some $\vec{v}_j$ (with $j > 1$) is a linear combination of the preceding vectors $\vec{v}_1, ..., \vec{v}_{j-1}$.

Use the proof structure taught in class, in the videos, and in the class notes.