Name: ______________________________
Final Assessment
Math& 264: Multivariable Calculus

Instructions: Please carefully complete these questions by hand. Be sure to show all work including sketching relevant graphs and providing exact answers.

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable. WRITE YOUR NAME ON EVERY PAGE

Upload your solutions to Gradescope by 10 am on Tuesday (12/15). During your presentation time, you will be asked to explain your thought process and reasoning on two randomly assigned question (one from part 1: new material, and one from part 2: cumulative material). Late submissions are available thru 5 pm on 12/17 with a 5% penalty (the penalty is fixed at 5% for the final).

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

https://docs.google.com/spreadsheets/d/17eCs-TpdpqMTu97_csL7NoFjqaO1Qqaj0IxjtMOqYM4/edit?usp=sharing

Reminders: It is okay to collaborate with peers and use online resources. However, the final work should be your own and you should be prepared to present on each question.

Part 1: New material

(1.1) Calculate the flux of $\vec{F} = \left\langle \cos z + xy^2, xe^{-z}, \sin y + x^2 z \right\rangle$ across the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$.

(1.2) Verify the Divergence Theorem for the vector field $\vec{F} = \langle z, y, x \rangle$ on the solid ball $x^2 + y^2 + z^2 \le 9$

(1.3) Calculate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \left\langle 3xy^2, xe^z, z^3 \right\rangle$ and *S* is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -3$ and $x = 1$.

(1.4) Find the circulation of $\vec{F} = \left\langle yz, 5xz, e^{xy} \right\rangle$ along the path $x^2 + y^2 = 16; z = 1$ which is oriented counterclockwise when viewed from above.

(1.5) Verify Stokes' Theorem for the vector field $\vec{F} = \langle y, z, x \rangle$ and the hemisphere $x^2 + y^2 + z^2 = 1$; $y \geq 0$ oriented in the direction of the positive *y*-axis.

(1.6) Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \langle xyz, xy, x^2yz \rangle$ and *S* consists of the top and four sides (but not the bottom) of the cube with vertices $(\pm 9, \pm 9, \pm 9)$, oriented outward.

(1.7) The temperature at the point (x, y, z) in a substance with conductivity $K = 8.5$ is $u(x, y, z) = 5y^2 + 5z^2$. Find the rate of heat flow inward across the cylindrical surface $y^2 + z^2 = 7$, $0 \le x \le 5$.

(1.8) Find the flux of $\vec{F} = \langle 0, y, -z \rangle$ across the closed surface consisting of the paraboloid $y = x^2 + z^2$, $0 \le y \le 1$ and the disk $x^2 + z^2 \le 1, y = 1$.

(1.9) Evaluate the surface integral $\iint_S x^2 z^2 dS$ where *S* is the part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 4$.

(1.10) Find the area of the part of the surface of $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

Part 2: Cumulative (review) material

(1.11) Find the length of the curve $r = \sqrt{1+\cos 2\theta}$ on $0 \le \theta \le \pi\sqrt{2}$.

(1.12) Find the area of the region inside the circle $r = 6$ and above the line $r = 3\csc\theta$.

(1.13) Test the series $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$ for convergence or divergence. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.[1]

(1.14) Test the series $\sum_{n=3}^{\infty} \frac{5n^3+3n}{n^2(n-2)(n^2-5)}$ for convergence or divergence. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.

[1] For tests requiring it, you do not need to verify that sequences are decreasing. This holds true throughout the Assessment.

(1.15) Calculate $\oint_C (6y+x)dx+(y+2x)dy$ where *C* is the circle $(x-2)^2+(y-3)^2=4$ traversed counterclockwise.

(1.16) Find the average distance from a point $P(x,y)$ in the disk $x^2+y^2 \le a^2$ to the origin

(1.17) Find the work done by the force $\vec{F} = \langle xy, y - x \rangle$ over the straight line from (1,1) to (2,3).

(1.18) Let $\vec{F} = \nabla\left(x^3 y^2\right)$ and let *C* be the path in the *xy*-plane along line segments from (-1,1) to (0,0) to (1,1). Evaluate the work in two ways.

(1.19) Set-up triple integrals for the volume of the sphere with radius 2 in spherical, cylindrical, and rectangular coordinates.

(1.20) In what have take-home assessments and presentations:

a.) Helped you grow as a mathematician?

b.) Helped/hurt your chances of being successful this quarter?

c.) Helped/hurt your chances of future mathematical success?

d.) And how many hours did you spend working on this final assessment?