Name: Key.

Assessment 7

Math& 264: Multivariable Calculus

Instructions: Please carefully complete these questions by hand. Be sure to show all work including sketching relevant graphs and providing exact answers.

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable. WRITE YOUR NAME ON EVERY PAGE

Upload your solutions to Gradescope by 9 am on Monday (11/16). During your presentation time, you will be asked to explain your thought process and reasoning on one randomly assigned question. Late solutions are available thru 2 pm with a 5% penalty (smaller penalties if you can document that this is a revision and only minor changes).

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

https://docs.google.com/spreadsheets/d/17eCs-TpdpqMTu97_csL7NoFjqaO1Qqaj0lxjtMOqYM4/edit?usp=sharing

Reminders: Show all steps (**including in evaluating limits**). Clearly state conclusions. Distinguish between sequences (lists) and series (sum of lists). It is okay to collaborate with peers and use online resources. However, the final work should be your own and you should be prepared to present on each question.

(1.1) Test the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\sqrt{n}}{3n+7}$ for convergence or divergence. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.[1]

$\lim_{n \to \infty} \frac{\sqrt{n}}{3n+7} = 0$ so the series converges by the Ast.

$\lim_{n \to \infty} \frac{\frac{\sqrt{n}}{3n+7}}{\frac{1}{\sqrt{n}}} = \lim_{n \to \infty} \frac{n}{3n+7} = \frac{1}{3}$ since $\sum \frac{1}{\sqrt{n}}$ diverges, this series does not converge absolutely by the LCT.

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{3n+7}$ converges conditionally by the alternating series test.

[1] For tests requiring it, you do not need to verify that sequences are decreasing. This holds true throughout the Assessment.

(1.2) Test the series $\sum_{k=1}^{\infty} 4ke^{-k}$ for convergence or divergence. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.

$$\lim_{k\to\infty}\left|\frac{4(k+1)e^{-(k+1)}}{4ke^{-k}}\right|$$

$$= \lim_{k\to\infty} \frac{4(k+1)e^{k}}{4ke^{k+1}}$$

$$= \lim_{k\to\infty} \frac{4(k+1)}{4ke}$$

$$= \frac{1}{e} < 1$$

$\therefore$ The series converges absolutely by the ratio test.

(1.3) Test the series $\frac{1}{2}+\frac{1\cdot5}{2\cdot5}+\frac{1\cdot5\cdot9}{2\cdot5\cdot8}+\frac{1\cdot5\cdot9\cdot13}{2\cdot5\cdot8\cdot11}+\ldots$ for convergence or divergence. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.

$\frac{1}{2}+\frac{1\cdot5}{2\cdot5}+\frac{1\cdot5\cdot9}{2\cdot5\cdot8}+\ldots > \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\ldots$ which diverges by the test for divergence.

$\therefore$ The series diverges by the comparison test

(1.4) Test the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n^7}$ for convergence. If the series is convergent, use the alternating series estimation theorem to determine the minimum number of terms we need to add in order to find the sum with an error less than 0.00005.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n^7} = \underbrace{\sum_{n=1}^{k} \frac{(-1)^{n-1}}{2n^7}}_{S_k} + \underbrace{\sum_{n=k+1}^{\infty} \frac{(-1)^{n-1}}{2n^7}}_{R_k}$$

$$|R_k| < a_{k+1} = \frac{1}{2(k+1)^7}$$

we want $\frac{1}{2(k+1)^7} < 0.00005$

$$\Rightarrow \underbrace{\sqrt[7]{\frac{1}{2(0.00005)}} - 1}_{\approx 2.73} < k$$

so we need 3 terms to estimate the series w/in 0.00005.

A.S.T.

$$\lim_{n\to\infty} \frac{1}{2n^7} = 0 \quad \therefore$$ The series converges (absolutely) by the LCT.

L.C.T.

$$\lim_{n\to\infty} \frac{\frac{1}{2n^7}}{\frac{1}{n^7}} = \frac{1}{2}.$$

Since $\sum \frac{1}{n^7}$ is a convergent p-series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n^7}$ converges absolutely.

(1.5) Determine whether the series $\sum_{n=2}^{\infty}\left(\frac{-3n}{n+1}\right)^{4n}$ is convergent or divergent. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.

Root test

$$\lim_{N\to\infty} \sqrt[N]{\left|\left(-\frac{3N}{N+1}\right)^{4N}\right|}$$

$$= \lim_{N\to\infty} \left(\frac{3N}{N+1}\right)^{4}$$

$$= 81 > 1$$

$\therefore$ The series diverges by the root test.

(1.6) Test the series $\sum_{n=0}^{\infty}\frac{(-7)^n}{(2n+1)!}$ for convergence or divergence. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.

Ratio test.

$$\lim_{N\to\infty}\left|\frac{\frac{(-7)^{N+1}}{(2(N+1)+1)!}}{\frac{(-7)^N}{(2N+1)!}}\right|$$

$$= \lim_{N\to\infty}\frac{7^N\cdot 7\,(2N+1)!}{7^N(2N+3)!}$$

$$= \lim_{N\to\infty}\frac{7}{(2N+3)(2N+2)}$$

$$= 0 < 1$$

$\therefore$ The series converges absolutely by the ratio test.

(1.7) Test the series $\sum_{n=1}^{\infty} \frac{(-1)^n 7n^n}{n!}$ for convergence or divergence. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.

Ratio Test

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} 7(n+1)^{n+1}}{(n+1)!}}{\frac{(-1)^n 7n^n}{n!}} \right|$$

$$= \lim_{n\to\infty} \frac{7(n+1)(n+1)^n n!}{7(n+1)n!\, n^n}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$$

$$= \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$$

$$= e^{\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right)}$$

$$= e^{\lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}}$$

$$\overset{H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\cdot -\frac{1}{x^2}}{-\frac{1}{x^2}}}$$

$$= e > 1$$

∴ The series converges absolutely by the ratio test

(1.8) Determine whether the series $\sum_{n=1}^{\infty} \left(\frac{n^2+8}{7n^2+6}\right)^n$ is convergent or divergent. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.

Root test

$$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{n^2+8}{7n^2+6}\right)^n\right|}$$

$$= \lim_{n\to\infty} \frac{n^2+8}{7n^2+6}$$

$$= \frac{1}{7} < 1$$

∴ The series converges absolutely by the root test.

(1.9) Determine whether the Ratio Test is inconclusive, conclusively convergent, or conclusively divergent for each of the given series. Based upon this example, what are some tips that will help you know when to use (or not use) the Ratio Test?

a.) $\sum_{n=2}^{\infty} \frac{5}{n^3}$

$\lim_{n\to\infty} \left| \frac{5}{(n+1)^3} \cdot \frac{n^3}{5} \right| = 1$ The ratio test is inconclusive

b.) $\sum_{n=2}^{\infty} \frac{n}{6^n}$

$\lim_{n\to\infty} \left| \frac{n+1}{6^{n+1}} \cdot \frac{6^n}{n} \right| = \lim_{n\to\infty} \frac{n+1}{6n} = \frac{1}{6} < 1$

The series converges absolutely by the ratio test.

c.) $\sum_{n=2}^{\infty} \frac{(-5)^{n-1}}{\sqrt{n}}$

$\lim_{n\to\infty} \left| \frac{(-5)^n}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(-5)^{n-1}} \right| = \lim_{n\to\infty} \frac{5\sqrt{n}}{\sqrt{n+1}} = 5 > 1$

The series diverges by the ratio test

d.) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$

$\lim_{n\to\infty} \left| \frac{\sqrt{n+1}}{1+(n+1)^2} \cdot \frac{1+n^2}{\sqrt{n}} \right| = 1$ The ratio test is inconclusive.

The ratio test works w/ an exponent of n,
The ratio test fails when the terms are rational (or the equivalent w/ roots).

(1.10) Determine whether the series $\sum_{n=1}^{\infty} 7\left(1+\frac{1}{n}\right)^{n^2}$ is convergent or divergent. If it converges, does the series converge conditionally or absolutely? Be sure to justify your work.

Root test

$$\lim_{n\to\infty} \sqrt[n]{\left|7\left(1+\frac{1}{n}\right)^{n^2}\right|}$$

$$= \lim_{n\to\infty} \cancel{\sqrt[n]{7}}^{\,1} \left(1+\frac{1}{n}\right)^{n}$$

$= e > 1$ (see #7 for details)

$\therefore$ The series diverges by the root test