Name: Key

Assessment 5

Math& 264: Multivariable Calculus

Instructions: Please carefully complete these questions by hand. Be sure to show all work including sketching relevant graphs and providing exact answers.

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable. WRITE YOUR NAME ON EVERY PAGE

Upload your solutions to Gradescope by 9 am on Monday (11/2). During your presentation time, you will be asked to explain your thought process and reasoning on one randomly assigned question. Late solutions are available thru 2 pm with a 5% penalty (smaller penalties if you can document that this is a revision and only minor changes).

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

https://docs.google.com/spreadsheets/d/17eCs-TpdpqMTu97_csL7NoFjqaO1Qqaj0IxjtMOqYM4/edit?usp=sharing

Pro tip: Make sure to check out the honors videos … and consider creating your own.

(1.1) Find the values of x for which the series $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ converges and the sum of the series for those values of x.

$\left(\frac{x-1}{3}\right)^n$

geometric series.

$a = 1$

$r = \frac{x-1}{3}$

$$\sum_{n=0}^{\infty} \frac{(x-1)}{3^n} = \frac{1}{1-\frac{x-1}{3}} = \frac{3}{3-x+1} = \frac{3}{4-x}$$

converges when

$\left|\frac{x-1}{3}\right| < 1$

$\Rightarrow |x-1| < 3$

$\Rightarrow -3 < x-1 < 3$

$\Rightarrow -2 < x < 4$

(1.2) Determine whether the sequence $a_n = \dfrac{2(\ln n)^2}{5n}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \frac{2(\ln n)^2}{5n} = \lim_{x\to\infty} \frac{2(\ln x)^2}{5x}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{4\ln x}{5x}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{4}{5x}$$

$$= 0$$

The sequence converges.

(1.3) Determine whether the sequence $\left\{\dfrac{(8n-1)!}{(8n+1)!}\right\}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \frac{(8n-1)!}{(8n+1)!} = \lim_{n\to\infty} \frac{\cancel{(8n-1)!}}{(8n+1)(8n)\cancel{(8n-1)!}}$$

$$= \lim_{n\to\infty} \frac{1}{(8n+1)(8n)}$$

$$= 0$$

(1.4) Determine whether the series $\sum_{n=1}^{\infty} \arctan(4n)$ converges or diverges. If it is convergent, find the sum of the series.

$$\lim_{n\to\infty} \arctan(4n) = \frac{\pi}{2}$$

since $\lim_{n\to\infty} \arctan(4n) = \frac{\pi}{2} \neq 0$

The series $\sum_{n=1}^{\infty} \arctan(4n)$ diverges by the test for divergence.

(1.5) Determine whether the sequence $a_n = \frac{(-2)^n}{7n!}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \frac{(-2)^n}{7n!} = \lim_{n\to\infty} \frac{(-1)^n}{7} \frac{2}{n} \cdot \frac{2}{(n-1)}, \dots, \underbrace{\frac{2}{4}, \frac{2}{3}, \frac{2}{2}, \frac{2}{1}}_{<1}$$

$\Rightarrow$ for $n \geq 4$

$$-\frac{2}{7n} < \frac{(-2)^n}{7n!} < \frac{2}{7n}$$

AND $\lim_{n\to\infty} -\frac{2}{7n} = \lim_{n\to\infty} \frac{2}{7n} = 0$

so the sequence converges by the squeeze thm.

(1.6) Determine whether the series $\sum_{n=1}^{\infty}\left(\frac{6}{e^n} + \frac{3}{n(n+1)}\right)$ converges or diverges. If it is convergent, find the sum of the series.

$$\sum_{n=1}^{\infty}\left(\frac{6}{e^n} + \frac{3}{n(n+1)}\right) = \sum_{n=1}^{\infty} 6 \cdot \left(\frac{1}{e}\right)^n + \sum_{n=1}^{\infty} \frac{3}{n(n+1)}$$

$$= \frac{\frac{6}{e}}{1-\frac{1}{e}} + \sum_{n=1}^{\infty}\left(\frac{3}{n} - \frac{3}{n+1}\right)$$

$$= \frac{6}{e-1} + \left(\frac{3}{1} - \cancel{\frac{3}{2}} + \cancel{\frac{3}{2}} - \cancel{\frac{3}{3}} + \cancel{\frac{3}{3}} - \cancel{\frac{3}{4}} + \dots\right)$$

$$= \frac{6}{e-1} + 3$$

$$\frac{3}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \Rightarrow 3 = A(n+1) + Bn \Rightarrow \begin{array}{l} n=0 \Rightarrow A=3 \\ n=-1 \Rightarrow B=-3 \end{array}$$

(1.7) Determine whether the series $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{4}{n}\right)$ converges or diverges. If it is convergent, find the sum of the series.

Suppose $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{4}{n}\right)$ converged.

$$\Rightarrow \sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{4}{n}\right) - \underbrace{\sum_{n=1}^{\infty}\frac{1}{2^n}}_{\text{convergent geometric series.}} = \sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{4}{n}-\frac{1}{2^n}\right)$$

$$= \sum_{n=1}^{\infty}\frac{4}{n}$$

$$= 4\sum_{n=1}^{\infty}\frac{1}{n} \text{ converges.}$$

But the harmonic series diverges $\Rightarrow\Leftarrow$

$\therefore$ The series diverges.

(1.8) Determine whether the series $\sum_{n=1}^{\infty}\left((0.4)^{n-1}-(0.3)^n\right)$ converges or diverges. If it is convergent, find the sum of the series.

$$= \sum_{n=1}^{\infty} 0.4^{n-1} - \sum_{n=1}^{\infty} 0.3^{n}$$

$a = 1$ $\quad$ $a = 0.3$

$r = 0.4$ $\quad$ $r = 0.3$

$$= \frac{1}{1-0.4} - \frac{0.3}{1-0.3}$$

$$= \frac{5}{3} - \frac{3}{7}$$

$$= \frac{26}{21}$$

(1.9) Express the number 2.7182 as a ratio of integers. Please show all steps.

$$2.71\overline{82} = 2.71 + 0.0082 + 0.000082 + \ldots$$

$\uparrow$

a and $r = \frac{1}{100}$

$$= \frac{271}{100} + \frac{0.0082}{1 - \frac{1}{100}}$$

$$= \frac{271}{100} + \frac{41}{4950}$$

$$= \frac{26911}{9900}$$

(1.10) Choose a non-zero rational number with a terminating decimal and show that it can be written in more than one way.

$$2.\overline{9} = 2 + \frac{9}{10} + \frac{9}{100} + \ldots$$

$$= 2 + \frac{\frac{9}{10}}{1 - \frac{1}{10}}$$

$$= 3$$

so the terminating decimal $3. = 2.\overline{9}$