10,8 Choosing a Convergence Test Math 264

Strategy for Testing Series

Let's do a brief review of all the tests we have introduced for determining convergence and how to decide which test to use.

To do this, remember the 4 known series:

P Geometric series:
$$\sum_{n=0}^{\infty} ar^n \begin{cases} converges & \text{if } |r| < 1 \\ diverges & \text{if } |r| \ge 1 \end{cases}$$

▶ P-series:
$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} converges & if \quad p > 1 \\ diverges & if \quad p \le 1 \end{cases}$$

Let
$$\sum_{n=1}^{\infty} a_n$$
 be given.

The alternating harmonic series 5 (-1) "+1 converges (conditionally)

For Positive Series:

TEST FOR DEVERGINE.
The Divergent Test (nth-Term Test): Always check this test first.

$$\lim_{n \to \infty} a_n \begin{cases} = 0 & then & inconclusive! \\ \neq 0 & then & \sum_{n=1}^{\infty} a_n & diverges \end{cases}$$

The Direct Comparison Test: Consider whether dropping terms in the numerator or

denominator gives a series that we know converges or diverges, $\sum b_n$.

Pro-tip: If using a comparison test, remember that you need to specifically state whether the

If
$$a_n \leq b_n$$
 and $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges

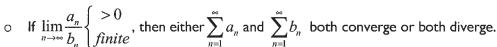
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series you are

(strong easy)

The Limit Comparison Test: Consider the dominant term in the numerator and comparing

to converges denominator, and come up with a series that we know converges or diverges, $\sum b_n$. or diverges.



o If
$$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$$
 and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

o If
$$\lim_{n\to\infty}\frac{a_n}{b_n}=0$$
 and $\sum_{n=1}^{\infty}b_n$ converges, then $\sum_{n=1}^{\infty}a_n$ converges.

O If $\lim_{n\to\infty}\frac{a_n}{b_n}=\infty$ and $\sum_{n=1}^{\infty}b_n$ diverges, then $\sum_{n=1}^{\infty}a_n$ diverges.

Description $\sum_{n=1}^{\infty}a_n$ diverges.

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The Ration Test. Best to use when there is a factorial or a constant to the power of n. Don't USE

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \begin{cases} <1 & then & absolutely-convergent \Rightarrow convergent \\ >1 & then & divergent \end{cases}$$

$$=1 & then & inconclusive!$$

(easy-weak)

The Root Test. Best to use when the terms have power n. Don't Use wil p-series,

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} \begin{cases} <1 & then & absolutely-convergent \Rightarrow convergent \\ >1 & then & divergent \\ =\infty & then & divergent \\ =1 & then & inconclusive! \end{cases}$$

(hard - weak)

The Integral Test. Best to use when the other tests fail. If $a_n = f(n)$ is decreasing, positive and continuous,

If
$$\begin{cases} b_{n+1} \leq b_n \\ \lim_{n \to \infty} b_n = 0 \end{cases}$$
 then
$$\sum_{n=1}^{\infty} a_n$$
 converges (either absolutely or conditionally).

Remirdes: Absolute Convergence. If $\sum_{n=0}^{\infty} a_n$ has some negative terms (maybe alternating or not), then

consider
$$\sum_{n=1}^{\infty} |a_n|$$
. If $\sum_{n=1}^{\infty} |a_n|$ converges, $\sum_{n=1}^{\infty} a_n$ is converging absolutely.

In short:

- 1. The nth-Term Test: Unless $a_n \rightarrow 0$, the series diverges.
- **2.** Geometric series: $\sum ar^n$ converges if |r| < 1; otherwise it diverges.
- **3.** p-series: $\sum \frac{1}{n^p}$ converges if p > 1; otherwise it diverges.
- 4. Series with nonnegative terms: Try the Integral Test, Ratio Test or Root Test. Try comparing to a known series with the Comparison Test or the Limit Comparison test.
- 5. Series with some negative terms: Does $\sum |a_n|$ converges? If yes, so does $\sum a_n$ since absolute convergence implies convergence.
- **6. Alternating series**: $\sum a_n$ converges if $\sum |a_n|$ is decreasing and $|a_n| \to 0$.

	EASY	HARD
WEAK	Test for Divergence Alternating Series Test Harmonic Series Alt. Harmonic Series	Telescoping series
POWERFUL	Geometric series P-series Limit companison test Ratio Test Root Test	Companison. Test Integral Test