**Name: ____________________________**
**Final Exam**
**Math& 264: Multivariable Calculus**

Instructions: Please carefully complete these questions by hand. Be sure to show all work including sketching relevant graphs, working carefully through limits, evaluating integrals, thoroughly working through convergence/divergence tests, and providing exact answers.

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable.

Upload your solutions to Gradescope by 9 am on the day of your presentation (6/8-10). During your presentation time, you will be asked to explain your thought process and reasoning on two randomly assigned questions.  Late solutions are available thru 2 pm on (6/10) with a 2 point penalty (smaller penalties if you can document that this is a revision the only incorporates minor changes).

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

https://docs.google.com/spreadsheets/d/1IGUR3J5qnXkbLUhWWeoMjbxYb1peN9lt5gZYj7wAtCo/edit?usp=sharing

(1.1) Find the limit of the sequence $\left\{ \frac{\ln n}{n^{1.3}} \right\}$ or determine that the sequence diverges.

(1.2) Evaluate the series $\sum_{n=0}^{\infty} e^{-4n}$ or state that it diverges.

(1.3) Determine the convergence or divergence of the series $\sum_{k=3}^{\infty} \frac{2}{k(\ln k)^2}$. If it converges, does it converge conditionally or absolutely?

(1.4) Determine whether the series $\sum_{k=1}^{\infty} \sqrt{\frac{k^3}{16k^5+3}}$ converges or diverges. If it converges, does it converge conditionally or absolutely?

(1.5) Determine whether the series $\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{\sqrt{k^6+4}}$ converges or diverges. If it converges, does it converge conditionally or absolutely?

(1.6) Determine whether the series $\sum_{k=1}^{\infty}\left(\frac{k}{k+3}\right)^{4k^2}$ converges or diverges. If it converges, does it converge conditionally or absolutely?

(1.7) Determine <u>ALL</u> values of *x* for which the series $\sum_{k=1}^{\infty}\frac{x^k}{7^k}$ converges.

(1.8) Use the methods developed in this class to find the exact area of the region bounded by all leaves of the rose $r = 4\cos 5\theta$.

(1.9) Rewrite the integral $\int_{0}^{5}\int_{-1}^{0}\int_{0}^{6x+6} dy\,dx\,dz$ in the order of integration $dz\,dx\,dy$ and then evaluate the resulting integral.

(1.10) Find the volume of the solid below the hyperboloid $z = 5 - \sqrt{1 + x^2 + y^2}$ and above the region $\left\{(r,\theta) : \sqrt{3} \le r \le \sqrt{15}, 0 \le \theta \le 2\pi\right\}$

(1.11) Find the mass of the solid paraboloid $D=\left\{(r,\theta,z):0\le z\le 100-r^2, 0\le r\le 10\right\}$ with density $\rho(r,\theta,z)=1+\frac{z}{100}$.

(1.12) Find the circulation about the path $\vec{r}(t) = \langle 5\sin t, 2\sin t, 4\cos t \rangle$ for $0 \le t \le 2\pi$ over the field $\vec{F} = \langle 2xy + z^2,\ x^2,\ 2xz \rangle$.

(1.13) Evaluate the line integral $\int_C \nabla\left(\frac{x^2+y^2+z^2}{2}\right)\cdot d\vec{r}$ along the oriented curve $\vec{r}(t)=\left\langle \cos t, \sin t, \frac{t}{\pi}\right\rangle$ for $\frac{\pi}{6}\le t\le\frac{3\pi}{2}$. Evaluate the integral directly and using the Fundamental Theorem for Line Integrals.

(1.14) Compute the circulation <u>and</u> outward flux over the field $\vec{F} = \nabla\left(\sqrt{x^2 + y^2}\right)$ and across the boundary of the region $R = \{(r,\theta): 2 \le r \le 7,\ 0 \le \theta \le \pi\}$.

(1.15) Compute the curl of the field $F = \left\langle 7xz^7e^{y^6},\, 6xz^7e^{y^6},\, 7xz^6e^{y^6} \right\rangle$.

(1.16) Evaluate the Jacobian $J\left(u,v,w\right)$ for the transformation $x = 2v + 2w, y = u + w, \text{ and } z = u + v$.

(1.17) Find the flux of $\vec{F} = \left\langle e^{-y},\ 2z,\ 4xy \right\rangle$ across the curved sides of the surface $S = \left\{ (x, y, z) : z = \cos y,\ |y| \le \pi,\ 0 \le x \le 4 \right\}$ with an upward pointing normal vector.

(1.18) Evaluate $\iint\limits_S \left(\nabla \times \vec{F}\right) \cdot \vec{n}\, dS$ where $\vec{F} = \left\langle x+y,\ y+z,\ z+x \right\rangle$, *S* is the tilted disk enclosed by $\vec{r}\left(t\right) = \left\langle 2\cos t,\ 3\sin t,\ \sqrt{5}\cos t \right\rangle$, and with $\vec{n}$ pointed in an upward direction.

(1.19) Fourier's Law of heat transfer (or heat conduction) states that the heat flow vector $\vec{F}$ at a point is proportional to the negative gradient of the temperature; that is, $\vec{F} = -k\nabla\vec{T}$, which means that heat energy flows from hot regions to cold regions. The constant *k* is called the conductivity (with units W/m-K). The temperature function for the region $D = \{(x, y, z) : 0 \le x \le 3,\ 0 \le y \le 5,\ 0 \le z \le 4\}$ is $T(x, y, z) = 100 - 5x - 2y + z$. Assume $k = 1$. Find the outward flux.