Name: ___________________________
Assessment 8
Math& 264: Multivariable Calculus

Instructions: Please carefully complete these questions by hand. Be sure to show all work including sketching relevant graphs and providing exact answers.

Should you choose to work these on scratch paper, please do not put more than one question on a page. Additional sheets of paper are acceptable.

Upload your solutions to Gradescope by 9 am on Monday (6/1). During your presentation time, you will be asked to explain your thought process and reasoning on one randomly assigned question. Late solutions are available thru 2 pm with a 2 point penalty (smaller penalties if you can document that this is a revision and only minor changes).

Please make sure to sign up for your presentation slot. If you are unavailable for any of the times available, please send me a note in Slack and we will find a time that works for you.

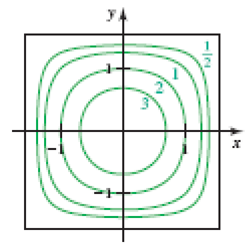
https://docs.google.com/spreadsheets/d/1IGUR3J5qnXkbLUhWWeoMjbxYb1peN9lt5gZYj7wAtCo/edit?usp=sharing

(1.1) Evaluate the line integral to calculate the (CCW) circulation of $\vec{F} = \left\langle 2xy^2, 3x^3 + y \right\rangle$ about the curve *C* which is the boundary of $\left\{ (x, y) : 0 \le y \le \sin x, 0 \le x \le \pi \right\}$.

(1.2) Consider the triangular region with vertices (0,0), (6,0), and (0,3) and the vector field $\vec{F} = \langle -3y, 5x \rangle$. Compute the two-dimensional divergence of the vector field and then evaluate both integrals in Green's Theorem and check for consistency (assume CCW orientation along the boundary).

(1.3) Compute the curl of the vector field $\vec{F} = \left\langle 6xz^6 e^{y^5}, 5xz^6 e^{y^5}, 6xz^5 e^{y^5} \right\rangle$

(1.4) An idealized two-dimensional ocean is modeled by the square $R=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\times\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ with boundary *C*. Consider the stream function $\psi(x,y)=4\cos x\cos y$ defined on *R* as shown in the figure. Find the total outward flux across *C* <u>and</u> the circulation on *C* assuming counterclockwise orientation.



(1.5) Assume that on $\mathbb{R}^2$, the vector field $\vec{F} = \langle f, g \rangle$ has a potential function φ such that $f = \varphi_x$ and $g = \varphi_y$, and it has a stream function ψ such that $f = \psi_y$ and $g = -\psi_x$. Show that the equipotential curves (level curves of φ) and the streamlines (level curves of ψ) are everywhere orthogonal.

(1.6) For the vector field $\vec{F} = \nabla\left(\sqrt{x^2+y^2}\right)$, compute the circulation <u>and</u> outward flux across the boundary of the region *R* which is the half annulus $\{(r,\theta): 2 \le r \le 7, 0 \le \theta \le \pi\}$.

(1.7) Calculate the divergence of the field $\vec{F} = \frac{\langle x, y, z \rangle}{x^2 + y^2 + z^2} = \frac{\vec{r}}{|\vec{r}|^2}$ and express the result in terms of the position vector $\vec{r}$ and its magnitude.

(1.8) Consider the rotational velocity field $\vec{v} = \langle -8y, 0, 2x \rangle$. If a paddle wheel is placed in the *xy*-plane with its axis normal to this plane, what is its angular speed? What about if it is placed in the *xz*-plane with its axis normal to this plane?

(1.9) Suppose a solid object in $\mathbb{R}^3$ has a temperature distribution given by $T(x,y,z)=100e^{-x^2+y^2+z^2}$.
The heat flow vector field in the object is $\vec{F}=-k\nabla T$, where the conductivity *k* is a property of the material. The divergence of the heat flow vector is the Laplacian of *T*: $\nabla\cdot F=-k\nabla\cdot\nabla T=-k\nabla^2 T$.
Compute the heat flow vector field for the temperature distribution.

(1.10) For the general rotation field $\vec{F}=\vec{a}\times\vec{r}$, where $\vec{a}=\langle a_1,a_2,a_3\rangle$ is a nonzero constant vector and $\vec{r}=\langle x,y,z\rangle$, show that $\operatorname{curl}(\vec{F})=2\vec{a}$.