Mathematics Through the Eyes of Faith
by Bradley and Howell

Four students at a Christian college, Juanita, an art major, Kristoff, a biology major, Alexandra, a mathematics major, and Tiquan, a religious studies major, are taking an introductory class together. Their professor has given them a collaborative project: they are to think about the relationship between mathematics and Christian belief. To get them started, she asked them a question: Could God have made a world in which $2 + 2 \neq 4$? Let's listen to their conversation.

JUANITA: So tell me again what our professor wants us to talk about.

KRISTOFF: She wants us to talk about the relationship between mathematics and Christian belief.

JUANITA: Math and our faith? This is going to be one short conversation.

TIQUAN: Well, let's see. She gave us a question to start us off: Could God have made a world in which $2 + 2 \neq 4$?

JUANITA: Sure. Let the symbol "4" stand for what we usually mean by "5." Nothing to it.

ALEXANDRA: No, that's not what she meant. Could God make $2 + 2 \neq 4$ where 2, +, =, and 4 have their usual meanings?

KRISTOFF: Well, why not? God can do anything—that's what omnipotence means, right?

TIQUAN: I don't think it's that simple. Suppose $2 + 2 = 5$. Doesn't that mean that any time anyone combines 2 things

pages 1-5, 9, 236-8

with 2 other things, a fifth thing has to mysteriously appear? I could put that to good use in my wallet!

JUANITA: You're saying there are things God can't do.

TIQUAN: Yes, I am. Scripture says that too. For instance, it says that God cannot be tempted by evil. That means he can't do evil. He can't lie. For instance, he can't decide to become an atheist.

KRISTOFF: Ha ha.

TIQUAN: No, I'm serious! I'm sure you've heard that old question, can God make a rock so heavy he can't lift it? No. God can't do things that don't make sense.

ALEXANDRA: As a math major, I'd say it differently—God can't do things that are logically inconsistent, like denying that $2 + 2 = 4$. In my view, $2 + 2$ has always equaled 4 and always will.

JUANITA: If math is eternal like God, is it as great as God?

ALEXANDRA: No, but I believe that even God can't make $2 + 2 = 5$.

JUANITA: No way! If God has to obey the laws of math, that makes math greater than God.

ALEXANDRA: Tiquan, you're the religious studies major. Help me out!

TIQUAN: There's a couple of ways to look at it. The laws of math could be part of God's nature. They're not greater than God; they're part of God—ideas in God's mind. But some people think that's claiming more about God than we can know. They would agree that God made the world and controls it. They would also agree that God understands everything there is about mathematics. But the question as to whether math is part of God, we don't know and can't know.

KRISTOFF: What was that first way—ideas in God's mind? God doesn't have a mind like we do!

TIQUAN: The phrase "God's mind" is a metaphor. It's just a way of saying that God has always known and understood mathematical ideas.

JUANITA: That seems right. But it doesn't mean God thinks exactly like us. That's not what you're saying, is it?

ALEXANDRA: I agree with Juanita. There's got to be ways we don't think like God. For instance, in my math classes, we never "know something"—that is, accept it as true—unless we can prove it. But God knows everything; he doesn't have to write a proof to know that a theorem is true.

KRISTOFF: I've heard some theologians say that, while we think differently than God, there are analogies between our thoughts and God's thoughts. For instance, God is consistent, and when we think consistently we share that trait with God. I'd say I love my family. My love is way smaller than God's, but I think it's like his in important ways.

ALEXANDRA: I understand. But our topic is God and math. Aren't we getting away from it?

KRISTOFF: I want to go back to your point about proofs in mathematics. We don't prove theorems in biology. But I still believe that what I learn in my biology classes is true.

JUANITA: That's a different kind of knowledge than Alexandra is talking about, isn't it? You discover what's true in biology by doing experiments and observations and gathering data. Mathematical knowledge doesn't depend on experiments or observations. For me, art is different. When I'm painting, my "truth" is much more intuitive and more personal.

ALEXANDRA: It seems to me that mathematics is special. You're an art major, Juanita. What is true for artists can vary from one person to another, even from one day to another. And according to you, Kristoff, if God had made the world differently, what is true would be different. But in math, theorems cannot *not* be true.

TIQUAN: I'm not sure you're right about that. How can you be so sure theorems are necessarily true? Scripture never makes any mathematical claims.

ALEXANDRA: I don't know. But it seems right to me. There's a lot of intuition in math. My preschool nephew understands

that there's no biggest number, so he even seems to have some intuition about infinity.

TIQUAN: Do you think God is the source of this intuition?

ALEXANDRA: I guess I do.

JUANITA: You seem to like math, Alex. But there's one thing I really hate about it: in every problem, there's always one right answer.

ALEXANDRA: That's what I like best about it!

JUANITA: Also it doesn't make sense to me why math works. I learned these rules for manipulating symbols back in high school algebra and it felt like I was being programmed to be some kind of computer. But scientists seem to think algebra has some connection with reality.

KRISTOFF: Einstein followed rules for manipulating mathematical symbols with his equations for general relativity and predicted that light rays would bend when passing a star. Nobody had ever observed that but when his prediction was tested, it turned out to be right!

ALEXANDRA: Like I said, mathematics is special.

JUANITA: You don't need to get snobby about it! If it's special, *you* didn't make it special.

ALEXANDRA: Oops. Sorry.

KRISTOFF: I don't know if math is special, but it seems to me that there is something important here. Look at the DNA molecule. It's a double helix, a mathematical figure! It seems like everywhere we look in the universe we find math, and not just any old math. It's subtle and beautiful, but if we work at it we can understand it.

JUANITA: Do you see that as pointing toward God?

KRISTOFF: I do. It makes more sense to me to say that than to say God wasn't involved. Besides, it seems purposeful. Mathematics helps us understand gravity, electricity, heat, and many other physical things. It also helps us understand probability, and now we have a much better understanding of heredity than people had two hundred years ago.

TIQUAN: And genetic engineers trying to clone people and create superhumans, and worse.

KRISTOFF: Okay, math can be misused, but can't any of God's gifts? Could we understand God's creation without math? Could we be stewards of it?

TIQUAN: I see your point, but a lot of people have looked at the mathematical structure of nature and concluded that the universe is a machine—that we can understand it all without bringing God into the conversation.

KRISTOFF: Sure, but I look at a snowcapped mountain and think, "Isn't God wonderful!" Somebody else looks at the same scene and doesn't see God at all. How we interpret what we see depends on what we believe.

JUANITA: I can see how a biology major can get excited about mathematics. But math and art? They're pretty far apart.

ALEXANDRA: Maybe not as far as you think. You've reminded me of something I puzzle about. Some of my professors are pure mathematicians and they often say that the reason they do math is because it's so beautiful. Their reasons have nothing to do with gravity or DNA molecules or curing diseases.

TIQUAN: Maybe you'd better explain to us what they mean by beautiful.

ALEXANDRA: I'd say order, symmetry, surprise, the ability to express a lot very briefly, things like that.

JUANITA: You missed color and contrast, but other than that, it's similar to what a visual artist might say.

TIQUAN: Do you think the visual arts point us toward God?

JUANITA: Artists used to say things like that a lot; they don't much anymore. But I still think it's true. And I agree with Kristoff—the beauty of nature can do that too. Of course, there is much more to art than beauty, just like there is much more in mathematics than beauty, I suppose.

ALEXANDRA: So maybe these qualities can point mathematicians to God also.

TIQUAN: Maybe. Well, I'm not sure we understand the relationship between mathematics and Christian belief any better than when we started, but I think we understand the issues better.

KRISTOFF: I think it's time to head back to class.

Summary based upon the dialogue.

7. ***Is mathematics discovered or invented?***

Alexandra believes that even God can't make $2 + 2 = 5$, so for her, $2 + 2 = 4$ is not invented. Is she right? Did the Greek mathematicians (whose work Euclid compiled in his *Elements*) invent geometry, or did they discover it? Similarly, did Isaac Newton and Gottfried Leibniz invent or discover calculus? Perhaps the answer is, "Some of it was invented and some discovered." But if mathematics is a combination of invention and discovery, how can we tell which aspects were invented and which discovered? That is, where do we draw the line between invention and discovery? These questions have been discussed since the time of Plato and Aristotle in the fourth century BC, and the answer is still not clear. This question is one of many that chapter 10 investigates.

This is a more sophisticated explanation of the debate about whether math is discovered or invented.

Mathematics: Discovered or Invented?

In its June, 2008 issue, the newsletter of the European Mathematical Society featured two articles on "The Question": Is mathematics discovered or invented? Both were by well-known authors, one by Reuben Hersh, a philosopher of mathematics, and one by Barry Mazur, a mathematician.

Hersh's answer is "invented." Platonism, writes Hersh, "says mathematical objects exist independently of our knowledge or activity, and mathematical truth is objective, with the same status as scientific truth about the physical world. This may be boiled down to the phrase 'out there.' That's where mathematical entities are, meaning, not 'in here.'" But the idea that mathematical objects exist "out there" is, for Hersh, false. He goes on to say that Platonism "is incompatible with the standard view of the nature of reality, as held by the majority of scientists and mathematicians: there is only one universe, one real world, which is physical reality, including its elaboration into the realm of living things, and elaborated from there into the realm of humankind with its social, cultural, and psychological aspects. I have argued that these social, cultural and psychological aspects of humankind are real, not illusory or negligible, and that the nature of mathematical truth and existence is to be understood in that realm, rather than in some other independent 'abstract' reality." So we can see that Hersh is a materialist and his "invented" answer expresses his understanding that mathematics is rooted in human social, cultural, and psychological activity.[50]

For Mazur, the question is intensely real but not as clear cut as it is for Hersh. He says, "If you engage in mathematics long enough, you bump into The Question, and it won't just go away." He acknowledges the "in here" versus "out there" distinction but insists that whichever option we choose, it must be grounded in the experience of doing mathematics. He writes, "The bizarre aspect of the mathematical experience—and this is

what gives such fierce energy to The Question—is that one feels (I feel) that mathematical ideas can be hunted down, and in a way that is essentially different from, say, the way I am currently hunting the next word to write to finish this sentence. One can be a hunter and gatherer of mathematical concepts, but one has no ready words for the location of the hunting grounds. Of course we humans are beset with illusions, and the feeling just described could be yet another. There may be no location." He does not take a position as to whether "in here" or "out there" is the correct answer, but he does add clarity to what is at stake.[51]

For Platonists, Mazur writes, "mathematics is the account we give of the timeless architecture of the cosmos. The essential mission, then, of mathematics is the accurate description, and exfoliation, of this architecture." He cautions parties arguing on both sides. The "out there" position, he points out, "has the curious effect of reducing some of the urgency of that staple of mathematical life: *rigorous proof.* Some mathematicians think of mathematical proof as the certificate guaranteeing trustworthiness of, and formulating the nature of, the building-blocks of the edifices that comprise our constructions. Without proof: no building-blocks, no edifice." So, he says, advocates of this position need to provide an adequate account of proof from their perspective. The "in here" position, he argues often uses irrelevant arguments and its proponents need to be more careful. For instance, arguing that mathematics is a human cultural activity has no relevancy to the question; descriptions of the Grand Canyon written by three people would be shaped by their cultures and personal psychology in different ways, but that wouldn't make the Grand Canyon a cultural phenomenon.[52]

the authors continue

Mazur concludes his article with a statement expressing his reaction when someone begins to influence him to think that The Question is "no big deal": "But someone who is not in love won't manage to definitively convince someone in love of the non-existence of eros; so this mood never overtakes me for long. Happily I soon snap out of it, and remember again the remarkable sense of independence—autonomy even—of mathematical concepts, and the transcendental quality, the uniqueness—and the passion—of doing mathematics. I resolve then that (Plato or Anti-Plato) whatever I come to believe about The Question, my belief must thoroughly respect and not ignore all this."[53]