Assessment 10
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Math 254

Name: key

When a philosopher says something that is true then it is trivial.
When he says something that is not trivial then it is false.

Carl Friedrich Gauss
1777-1855 (German mathematician)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

No work = no credit
No CAS Calculators

Warm-ups (1 pt each): $div\left(curl\left(\vec{F}\right)\right) = \underline{0}$ $\qquad (x-2)^4 = \underline{x^4} - 8x^3 + 24x^2 - 32x + 16$

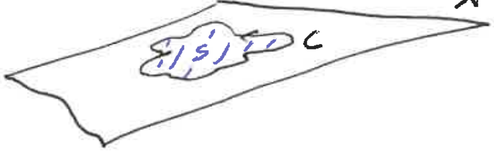
1.) (1 pt) Based upon the quote (above) how much value did Gauss place in the work of philosophers? Please explain using complete English sentences.

Gauss did not have much respect for philosophers.

2.) (12 pts) Let $\vec{F} = \left\langle -6y^2 + 6y, x^2 - 3z^2, -x^2 \right\rangle$. Use Stoke's Theorem to show that the work done by $\vec{F}$ along **any** simple closed curve contained in the plane $x + 2y + z = 1$ is equal to zero.

Note: This will require mathematical work (10 pts) and a little bit of writing (English) to explain your reasoning (2 pts).

$x + 2y + z = 1$



$$\text{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (-6y^2+6y) & (x^2-3z^2) & -x^2 \end{vmatrix}$$

$$\text{work} = \oint_C \vec{F} \cdot d\vec{r}$$

$$= \iint_S \text{curl}(\vec{F}) \cdot d\vec{S} \qquad = \langle 0 + 6z, -(-2x), 2x + 12y - 6 \rangle$$

sub in w/ parameters $\qquad = \langle 6z, 2x, 2x + 12y - 6 \rangle$

$$= \iint_S \langle 6(1-u-2v), 2u, 2u + 12v - 6 \rangle \cdot \langle 1, 2, 1 \rangle \, dA$$

parameterize S

$$= \iint_S 0 \, dA \qquad x = u$$

$$y = v$$

$$= 0 \qquad z = 1 - u - 2v$$

since the work is zero for an arbitrary path ~~closed~~ closed path C, the claim is proved.

$$\Rightarrow \vec{r}(u,v) = \langle u, v, 1 - u - 2v \rangle$$

$$\vec{r}_u = \langle 1, 0, -1 \rangle$$

$$\vec{r}_v = \langle 0, 1, -2 \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle 1, 2, 1 \rangle$$

3.) (20 pts) Let S be the closed surface that has two parts: The bottom face B is the unit disc in the xy-plane. The upper surface U is the paraboloid $z = 1 - x^2 - y^2$; $z \geq 0$. Let $\vec{F} = \langle x, y, z \rangle$.

a.) (10 pts) Find the flux of $\vec{F}$ across S by using the Divergence (Gauss') Theorem.

$$\text{Flux} = \iint_S \vec{F} \cdot d\vec{S}$$

$$= \iiint_E \operatorname{div} F \, dV$$

$$= \int_0^{2\pi} \int_0^1 \int_0^{1-r^2} 3r \, dz \, dr \, d\theta$$

$$= 2\pi \int_0^1 3r(1-r^2) \, dr$$

$$3r - 3r^3$$

$$= 2\pi \left[\frac{3}{2} r^2 - \frac{3}{4} r^4 \right]_0^1$$

$$= 2\pi \left(\frac{3}{2} - \frac{3}{4} \right) \text{ or } \frac{3}{2}\pi$$

b.) (10 pts) Calculate the flux of $\vec{F}$ across U. (The answer is around 4.75)

$$\vec{F}(R,\theta) = \langle R\cos\theta, R\sin\theta, 0 \rangle$$

$$\vec{r}_R = \langle \cos\theta, \sin\theta, 0 \rangle$$

$$\vec{r}_\theta = \langle -R\sin\theta, R\cos\theta, 0 \rangle$$

$$\vec{r}_R \times \vec{r}_\theta = \langle 0, 0, R \rangle$$

$$\text{Flux} = \iint_U \vec{F} \cdot d\vec{S}$$

$$= \iint_S F \cdot d\vec{S} - \iint_B \vec{F} \cdot d\vec{S}$$

B oriented down.

$$= \frac{3\pi}{2} - \iint_{R,\theta \text{ region}} \langle R\cos\theta, R\sin\theta, 0 \rangle \cdot \langle 0, 0, R \rangle \, dA$$

$$= \frac{3\pi}{2}$$