100	90's	80's	70's	60's	< 60
4	3	4	5	3	3

Assessment 9
Dusty Wilson
Math 254

$\bar{x} = 80.4\%$

med = 80.3%

Name: key

You are quite right in saying that it is well not to go brooding over one's own thoughts and feelings ... but you don't know what it is to live utterly alone.

George Stokes
1819-1903 (Irish mathematician)

No work = no credit
No CAS Calculators 1 3 3 1

Warm-ups (1 pt each): Expand $(3x-2)^3 = 27x^3 - 54x^2 + 36x - 8$ $\infty \cdot \infty =$ ∞

1.) (1 pt) The quote (above) is from a letter that Stokes wrote his future wife. Was Stokes completely satisfied with his mathematical endeavors? Please explain using complete English sentences.

No, Stokes was very lonely. Poor guy :(

2.) (10 pts) Calculate the flux of $\vec{F} = \langle y, z, 0 \rangle$ across the surface parameterized by $\vec{r}(u,v) = \langle u^3 - v, u+v, v^2 \rangle$ with $0 \le u \le 2$ and $0 \le v \le 3$ and the surface oriented downward.

$\vec{r}_u = \langle 3u^2, 1, 0 \rangle$

$\vec{r}_v = \langle -1, 1, 2v \rangle$

$\vec{r}_u \times \vec{r}_v = \langle 2v, -6u^2 v, 3u^2 + 1 \rangle$

Test orientation.
$\vec{r}(0,0) = \langle 0,0,0 \rangle$
$\vec{r}_u \times \vec{r}_v = \langle 0,0,1 \rangle$ @ $(0,0)$
which points up.

$$\text{Flux} = -\int_0^3 \int_0^2 \langle u+v, v^2, 0 \rangle \cdot \langle 2v, -6u^2 v, 3u^2+1 \rangle \, du\,dv$$

$$= -\int_0^3 \int_0^2 2uv + 2v^2 - 6u^2 v^3 \, du\,dv$$

$$= -\int_0^3 \left[u^2 v + 2uv^2 - 2u^3 v^3 \right]_{u=0}^{u=2} dv$$

$$= -\int_0^3 4v + 4v^2 - 16v^3 \, dv$$

$$= -\left[2v^2 + \tfrac{4}{3}v^3 - 4v^4 \right]_{v=0}^{v=3} \quad -(18 + 36 - 324) = 270$$

3.) (1 point extra credit) Last week I was late to class because I had to attend a training on how to use a super dishwasher (in the bio labs). The washer does a special rinse with Acid.

4.) (10 pts) Answer the following:

a.) If $f(x,y,z)=2x\sin(y)\ln(z)$, find ∇f.

$$\nabla f = \left\langle 2\sin y \ln z,\ 2x\cos y \ln z,\ \frac{2x\sin y}{z} \right\rangle$$

b.) Find the divergence of $\vec{F}=\langle e^{xy}, xy, z^4\rangle$ at the point $(1,0,2)$

$$\operatorname{div}\vec{F} = \vec{\nabla}\cdot\vec{F} = ye^{xy} + x + 4z^3$$

$$= 1 + 32$$

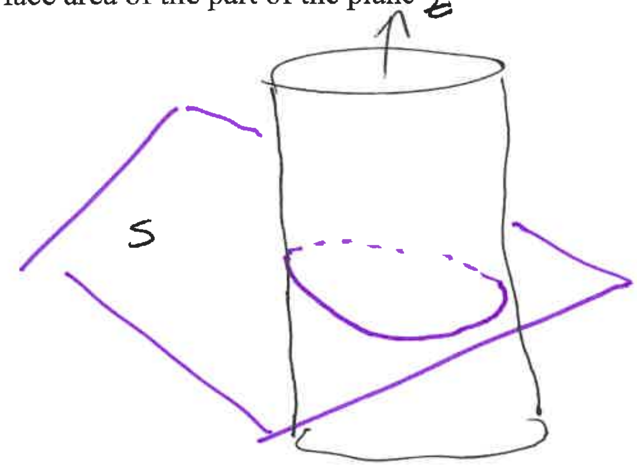
$$= 33$$

c.) Calculate the curl of $\vec{G}(x,y,z)=\langle xy, e^x, y+z\rangle$

$$\operatorname{curl}\vec{G} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & e^x & y+z \end{vmatrix}$$

$$= \langle 1, 0, e^x - x\rangle$$

5.) (10 pts) Use a surface integral to calculate the surface area of the part of the plane $x+y+z=5$ inside the cylinder $x^2+y^2=4$.



① parameterize S.

$x = u$

$y = v$ on $u^2+v^2 \le 4$

$z = 5-u-v$

$$\Rightarrow \vec{r}(u,v) = \langle u, v, 5-u-v\rangle$$

$$\vec{r}_u = \langle 1, 0, -1\rangle$$

$$\vec{r}_v = \langle 0, 1, -1\rangle$$

$$\Rightarrow \vec{r}_u \times \vec{r}_v = \langle 1, 1, 1\rangle$$

$$\Rightarrow |\vec{r}_u \times \vec{r}_v| = \sqrt{3}$$

② Integrate

$$SA = \iint_{u^2+v^2\le 4} \sqrt{3}\, dA = \sqrt{3}\cdot\pi\cdot 2^2 = 4\sqrt{3}\,\pi$$