Name: _____key_____

No work = no credit
No CAS Calculators

Warm-ups (1 pt each):   Centroid of the unit sphere
(assuming uniform density) = $(0,0,0)$ Expand $(2-x)^3 = \underline{8 - 12x + 6x^2 - x^3}$

1.) (1 pt) The quote by Manzi (above) is from the professor in the video this week. How does Manzi assess value in himself/others? Answer using complete English sentences.

Manzi believes we find value thru service to others

2.) (10 pts) Find the work done by the force field $\vec{F}(x,y) = \langle xy, y-x \rangle$ on a particle the moves along the line segment from $(1,1)$ to $(2,3)$.

$x = t$
$y = 2t - 1$
on $1 \le t \le 2$

$y = 2x - 1$
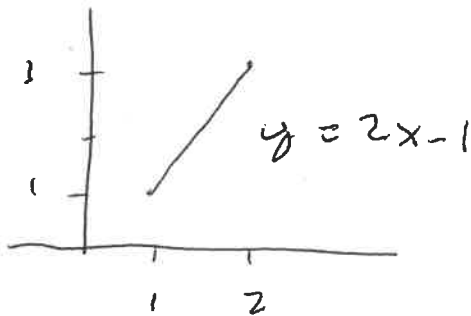
$$\text{Work} = \int_C \vec{F} \cdot d\vec{r}$$

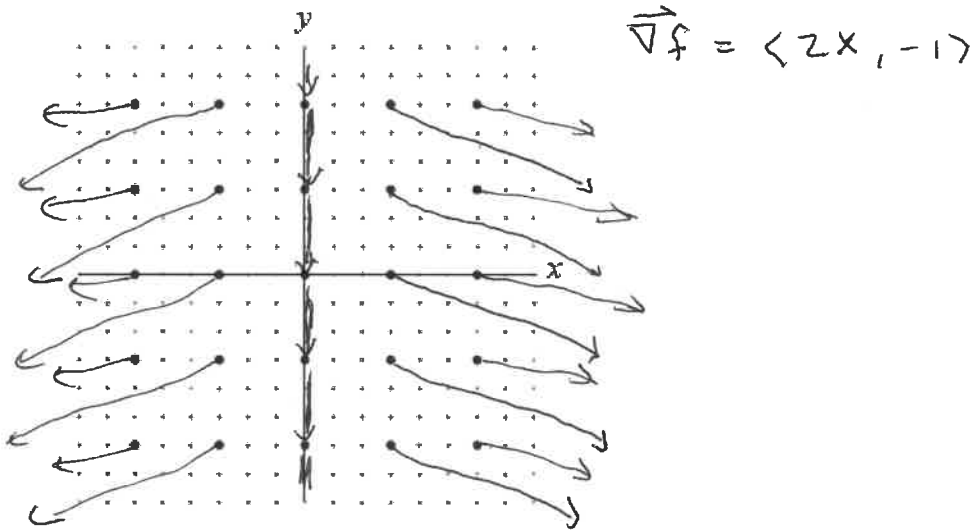$$= \int_1^2 [t(2t-1)\cdot 1 + (2t+1-t)\cdot 2] \, dt$$

$$= \int_1^2 (2t^2 - t + 2t - 2)\, dt$$

$$= \int_1^2 2t^2 + t - 2 \, dt$$

$$\text{Work} = \left[\frac{2}{3}t^3 + \frac{t^2}{2} - 2t\right]_1^2$$

$$= \frac{16}{3} + \frac{4}{2} - 4 - \left(\frac{2}{3} + \frac{1}{2} - 2\right)$$

$$= \frac{25}{6}$$

3.) (10 pts) Find the gradient vector field $\vec{\nabla f}$ of $f(x,y)=x^2-y$ and sketch it.

$$\vec{\nabla f} = \langle 2x, -1 \rangle$$

4.) (10 pts) Use a change of variables to evaluate $I = \iint_R (x-y)(2x+y)\,dA$ where R is the region bounded by $2x+y=4$, $2x+y=7$, $x-y=2$, and $x-y=-1$

Let $u = 2x+y$

$v = x-y$

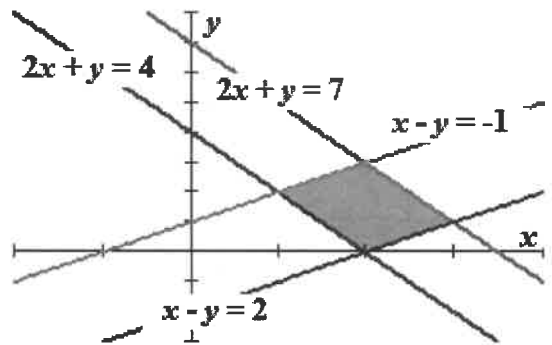$$\dfrac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= -3$$

scaling factor $= \dfrac{1}{3}$

$$I = \int_{v=-1}^{v=2} \int_{u=4}^{u=7} uv \cdot \dfrac{1}{3}\,du\,dv$$

$$= \dfrac{1}{6} \int_{-1}^{2} \left[u^2 v \right]_{u=4}^{7} dv$$

$$I = \dfrac{1}{6} \int_{-1}^{2} 33v\,dv$$

$$= \dfrac{33}{12} \left[v^2 \right]_{-1}^{2}$$

$$= \dfrac{33}{12} \cdot (4-1)$$

$$= \dfrac{33}{4}$$