Assessment 6
Dusty Wilson
Math 254

Name: Key

[Mathematics] is a kind of public language that allows us, as best we can, to try to achieve objectivity and certainty.

Javier Leach
1942 - 2016 (Prussian mathematician)

No work = no credit
No CAS Calculators

Warm-ups (1 pt each): Centroid of the unit sphere (assuming uniform density) = $(0,0,0)$ Expand $(x-2)^3 = x^3 - 2x^2 + 4x - 8$

1.) (1 pt) The quote by Leach (above) is from the author of our reading for this week. What does Leach mean when he calls math a, "public language?" Answer using complete English sentences.

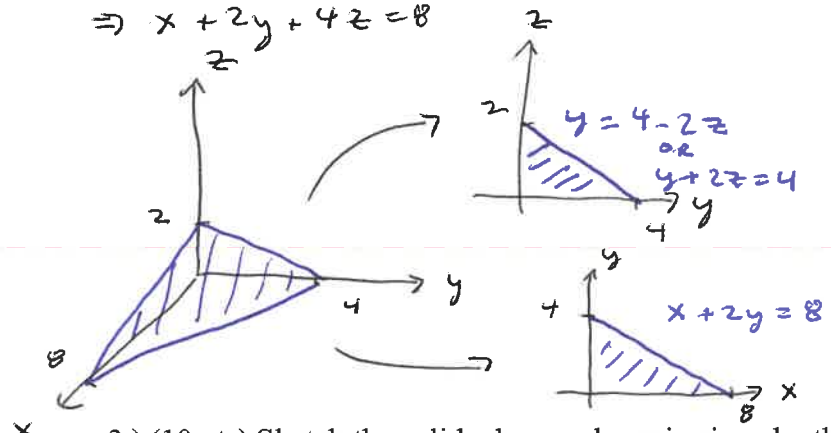
Math crosses geography, language, culture, and time.

2.) (10 pts) Rewrite the integral $\int_0^2 \int_0^{4-2z} \int_0^{8-2y-4z} f(x,y,z)\,dx\,dy\,dz$ as an equivalent iterated integral in the order $dz\,dy\,dx$.

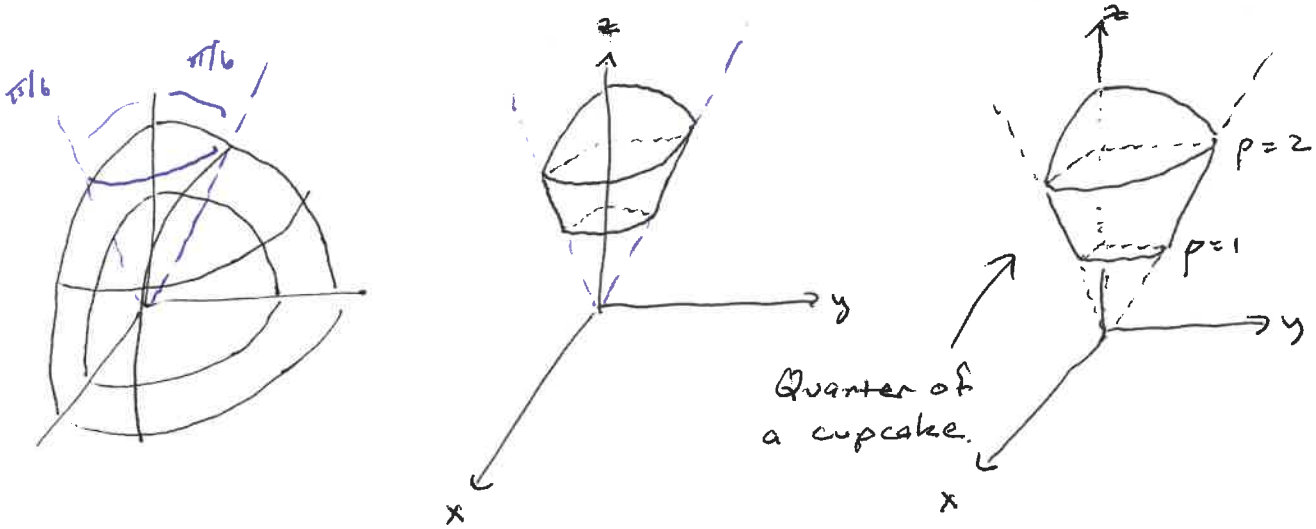
$x = 8 - 2y - 4z$
$\Rightarrow x + 2y + 4z = 8$

$y = \frac{1}{2}(8-x)z = \frac{1}{4}(8 - x - 2y)$

$$\int_{x=0}^{8} \int_{y=0}^{4-\frac{1}{2}x} \int_{z=0}^{\frac{1}{4}(8-x-2y)} f(x,y,z)\,dz\,dy\,dx$$

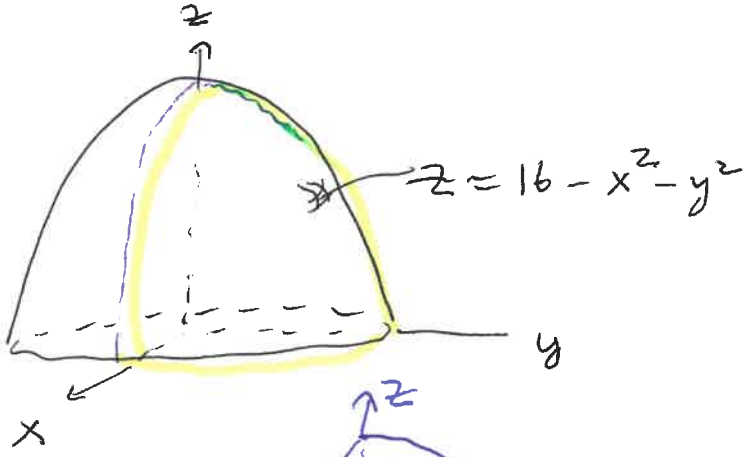


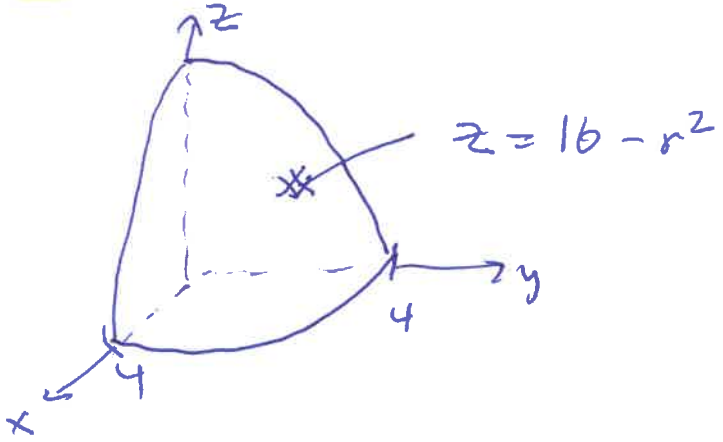
3.) (10 pts) Sketch the solid whose volume is given by the integral $\int_0^{\frac{\pi}{6}} \int_0^{\frac{\pi}{2}} \int_1^2 \rho^2 \sin\phi\,d\rho\,d\theta\,d\phi$



4.) (10 pts) Evaluate $\int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{16-x^2-y^2} \sqrt{x^2+y^2}\, dz\, dy\, dx$ by changing to cylindrical coordinates

$I =$





$$I = \int_0^{\pi/2} \int_0^4 \int_{z=0}^{z=16-r^2} r \cdot r \, dz \, dr \, d\theta$$

$$= \frac{\pi}{2} \int_0^4 \left[r^2 z \right]_{z=0}^{16-r^2} dr$$

$$= \frac{\pi}{2} \int_0^4 (16r^2 - r^4)\, dr$$

$$= \frac{\pi}{2} \left[\frac{16}{3} r^3 - \frac{r^5}{5} \right]_0^4$$

$$= \frac{1024}{15} \pi$$