$\bar{X} = 82.2\%$

med $= 80.3\%$

100	90's	80's	70's	60's	<60
3	5	3	6	5	0

Assessment 5
Dusty Wilson
Math 254

Name: Key

The infinite! No other question has ever moved so profoundly the spirit of man.

David Hilbert
1862 - 1943 (Prussian mathematician)

No work = no credit
No CAS Calculators

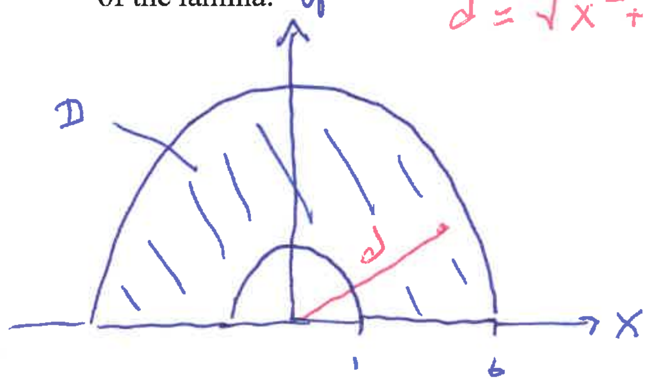
Warm-ups (1 pt each): Centroid of the unit circle (assuming uniform density) = $(0,0)$ Expand $(x-1)^3 = x^3 - 3x^2 + 3x - 1$

1.) (1 pt) The quote by Hilbert (above) is from the author of our reading for this week. According to Hilbert, what is the most profound question ever asked? Answer using complete English sentences.

The infinite boggles our finite minds.

2.) (10 pts) Consider the lamina bounded by the semicircles $y = \sqrt{1-x^2}$ and $y = \sqrt{25 - x^2}$ together with the portions of the x-axis that join them. The density of the lamina at a point is proportion to its distance from the origin. The mass of the lamina is 124π Find the centroid of the lamina.

$d = \sqrt{x^2+y^2}$



$\rho(x,y) = 3\sqrt{x^2+y^2}$

mass $= 124\pi$

$\bar{x} = 0$ (by symmetry).

we need $\bar{y} = \frac{M_x}{m}$

$M_x = \iint_D y\,\rho(x,y)\,dA$

rectangular coords.

$$M_x = \int_{-5}^{5} \int_{\begin{cases} 0, & -5 \le x \le -1 \\ \sqrt{1-x^2}, & -1 \le x \le 1 \\ 0, & 1 \le x \le 5 \end{cases}}^{\sqrt{25-x^2}} 3y\sqrt{x^2+y^2}\,dy\,dx$$

polar coords.

$$M_x = \int_0^{\pi} \int_1^5 3r\sin\theta \cdot r \cdot r\,dr\,d\theta$$

$$= \int_0^{\pi} \sin\theta\,d\theta \cdot \int_1^5 3r^3\,dr$$

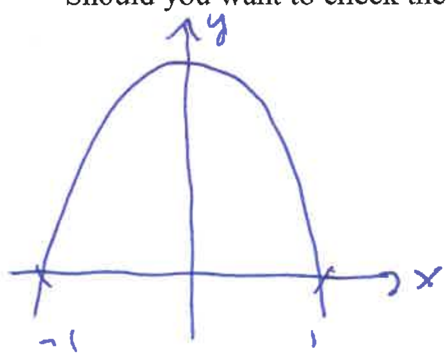
$$= 2 \cdot \left[\frac{3}{4}r^4\right]_1^5$$

$$= 936$$

$$\bar{y} = \frac{936}{124\pi}$$

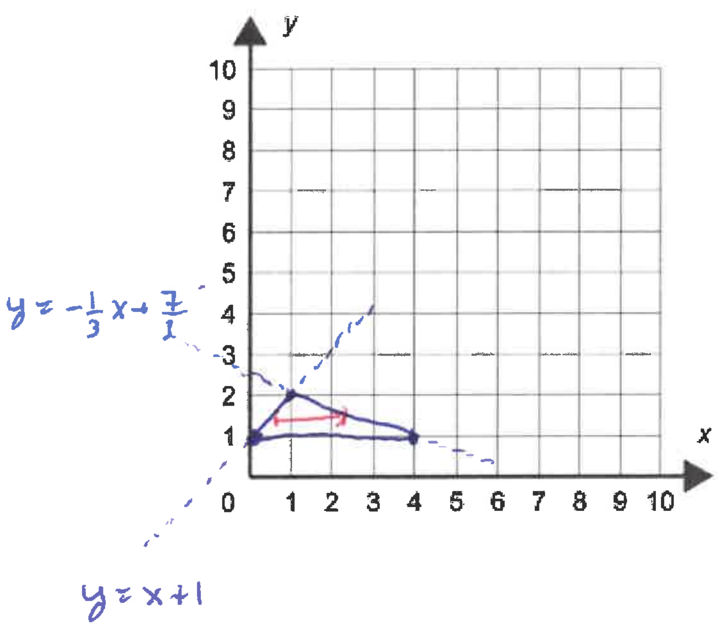
centroid $\left(0, \frac{936}{124\pi}\right)$

3.) (10 pts) **Set up** (do not evaluate) an iterated integral that represents the mass of the lamina occupies the region bounded by $y=1-x^2$ and $y=0$ and has density $\rho(x,y)=15ky$. Hint: Should you want to check the mass is $\underline{8k}$



$$m = \int_{-1}^{1}\int_{0}^{1-x^2} 15ky\,dy\,dx$$

4.) (10 pts) **Set up** an iterated integral to represent the moment about the x-axis of the triangular region with vertices at (0,1), (1,2), and (4,1) and that has density function $\rho(x,y)=9y$. Hint: Should you want to check the moment is 33.



$$M_x = \iint_D y\,\rho(x,y)\,dA$$

$$= \int_1^2 \int_{x=y-1}^{x=7-3y} y\cdot 9y\,dx\,dy.$$