Assessment 4
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Math 254

No work = no credit
No CAS Calculators

Name: Key

The enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious.

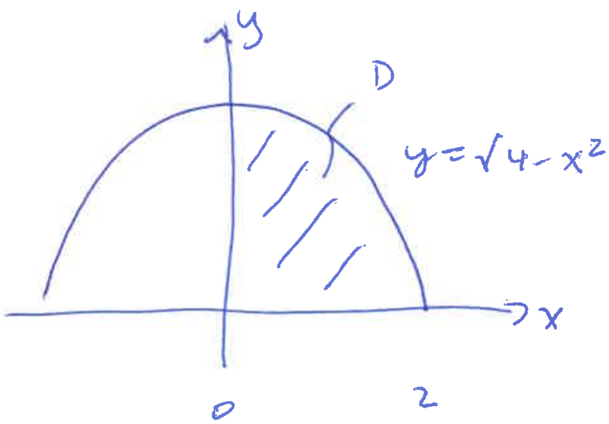
Eugene Wigner
1902 - 1995 (Hungarian Physicis)

Warm-ups (1 pt each): $\int_0^1\int_2^5 6\,dx\,dy = \underline{18}$ $\quad \int_3^4\int_1^1 \text{yuck}\,dx\,dy = \underline{0}$

1.) (1 pt) The quote by Wigner (above) is from our reading for this week. According to Wigner, how ought we to explain the usefulness of mathematics? Answer using complete English sentences.

Math's usefulness is mysterious.

2.) (10 pts) Evaluate the iterated integral $I = \int_0^2\int_0^{\sqrt{4-x^2}} x\,dy\,dx$ by converting to polar coordinates.



$$I = \int_0^{\pi/2}\int_0^2 r\cos\theta \cdot r\,dr\,d\theta$$

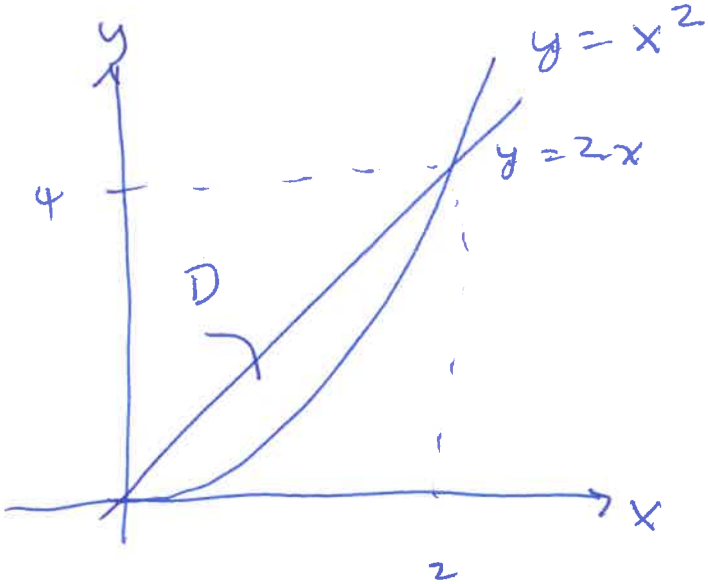
$$= \underbrace{\int_0^{\pi/2}\cos\theta\,d\theta}_{1} \cdot \int_0^2 r^2\,dr$$

$$= \left[\frac{r^3}{3}\right]_0^2$$

$$= \frac{8}{3}$$

$I =$

4.) (10 pts) Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x+2)\, dy\, dx$ and then evaluate the iterated integral.



$$I = \int_0^2 \left[4xy + 2y \right]_{y=x^2}^{y=2x} dx$$

$$= \int_0^2 ((8x^2 + 4x) - (4x^3 + 2x^2))\, dx$$

$$= \int_0^2 -4x^3 + 6x^2 + 4x\, dx$$

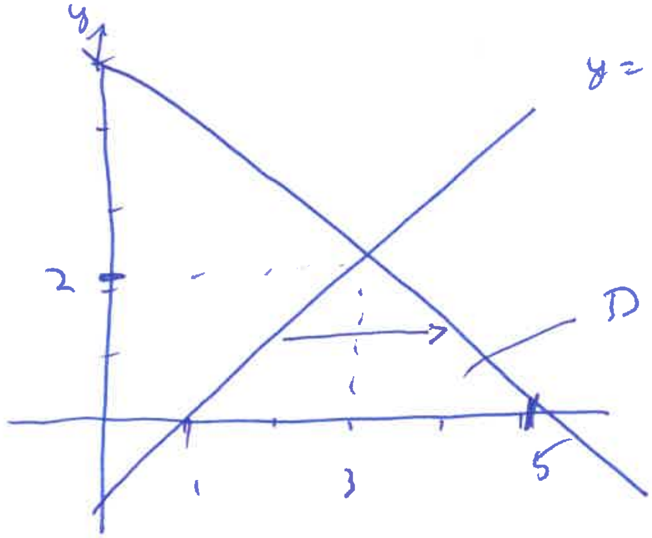
$$= \left[-x^4 + 2x^3 + 2x^2 \right]_0^2$$

$$= -16 + 16 + 8$$

$$= 8.$$

3.) (10 pts) In evaluating a double integral over D, the following result was found:

$I = \iint_D f(x,y)\,dA = \int_0^3 \int_0^{x-1} f(x,y)\,dy\,dx + \int_3^5 \int_0^{5-x} f(x,y)\,dy\,dx$, sketch D and express the double integral with reversed order of integration.

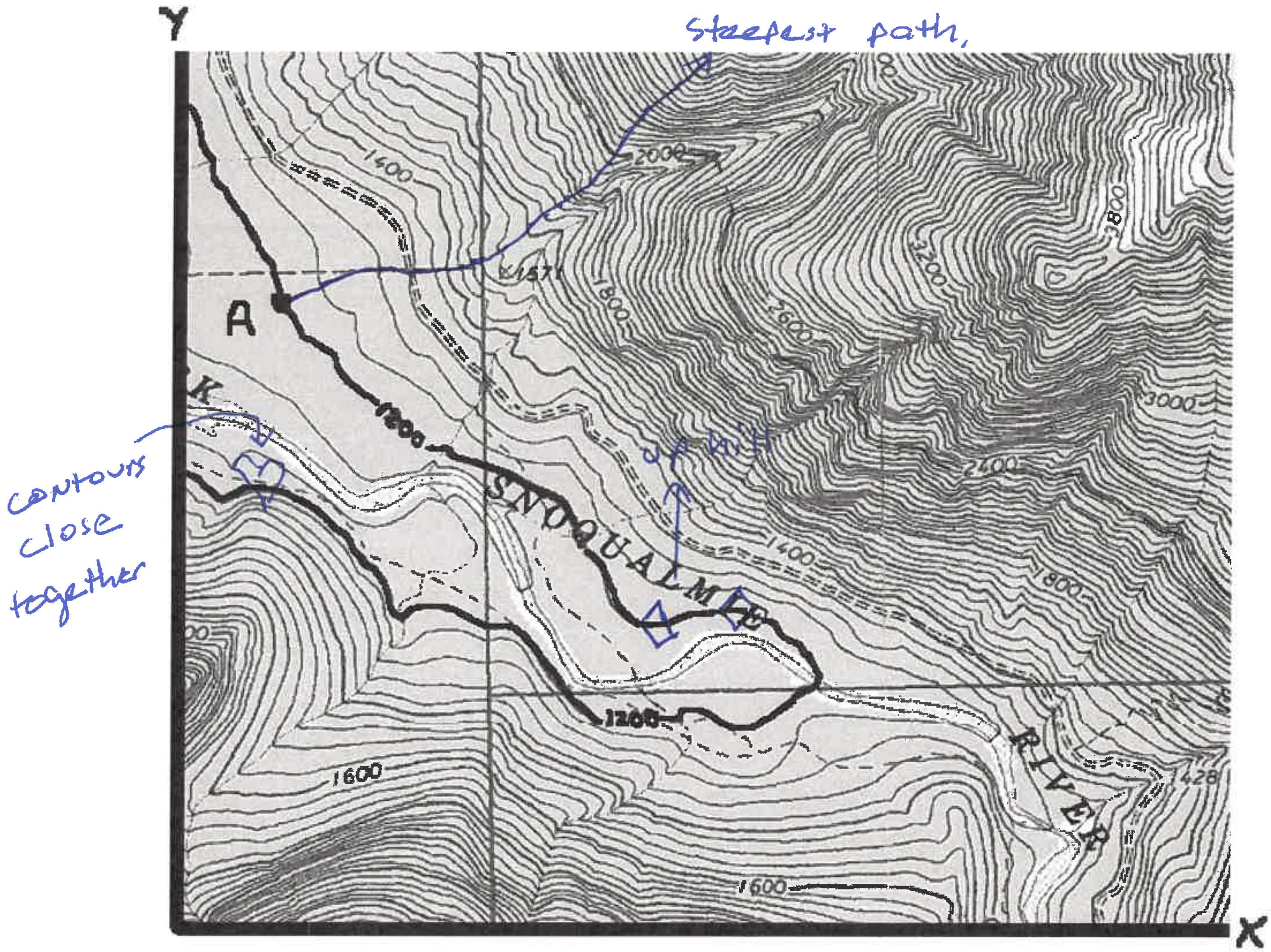


$y = x - 1 \Rightarrow x = y + 1$

$y = 5 - x \Rightarrow x = 5 - y$

so $I = \iint_D f(x,y)\,dA = \int_{y=0}^{y=2} \int_{x=y+1}^{x=5-y} f(x,y)\,dx\,dy$

5.) (3 bonus pts with no partial credit) Consider the contour plot (topographical map) of the valley through which the Snoqualmie River runs where $z = f(x, y)$ gives the altitude (in feet) at point (x, y) where x and y have the traditional orientation. The solid black line shows the level curve at 1,200 feet.



a.) On the contour plot, clearly mark with a diamond ♦ the point(s) of the level curve $f(x, y) = 1{,}200$ at which $f_x = 0$ and $f_y > 0$.

b.) On the contour plot, clearly mark with a heart ♥ the point(s) of the level curve $f(x, y) = 1{,}200$ at which the slope is greatest ($|\nabla f|$ is large).

c.) Beginning at point A, clearly sketch the path of steepest ascent.