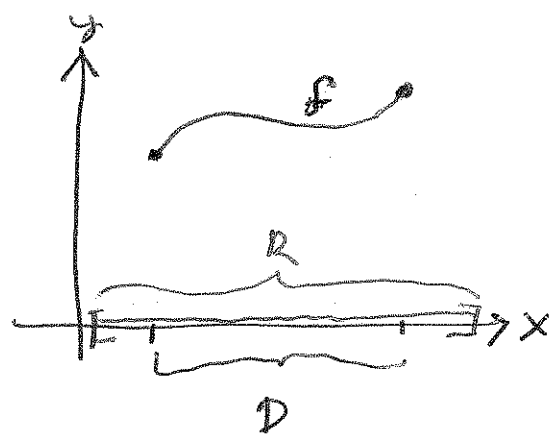


15.2: Double Integrals over General Regions.

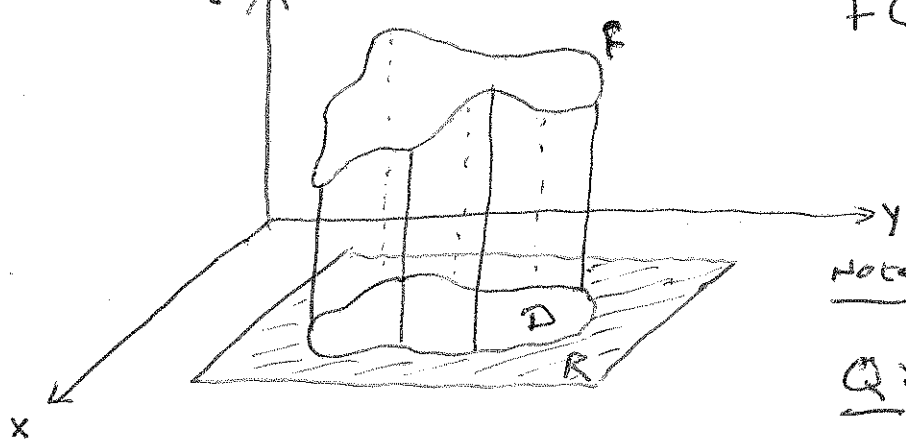
single integrals



$$F(x) = \begin{cases} f(x), & x \in D \\ 0, & x \in \mathbb{R} \text{ and } x \notin D \end{cases}$$

Note: D is bounded (otherwise we have an improper integral).

double integrals



$$F(x,y) = \begin{cases} f(x,y), & (x,y) \in D \\ 0, & (x,y) \in \mathbb{R} \text{ and } (x,y) \notin D \end{cases}$$

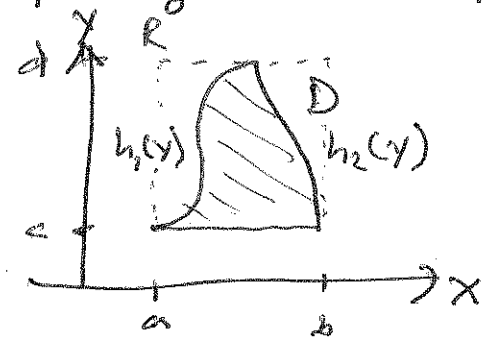
Note: D is bounded

Q: Is F cont?

★ Fubini's Thm Applies.

IF $D = \{(x,y) \mid c \leq y \leq d \text{ and } h_1(y) \leq x \leq h_2(y)\}$

That is:



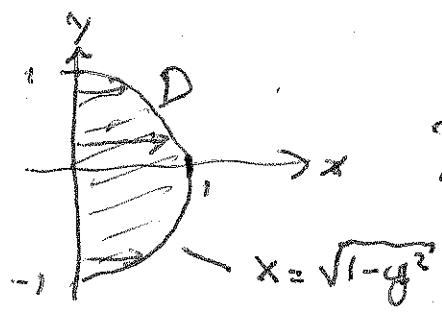
$$\begin{aligned}
 \iint_D F(x,y) dA &= \iint_R F(x,y) dA \\
 &= \int_c^d \int_a^b F(x,y) dx dy \\
 &= \int_c^d \int_{h_1(y)}^{h_2(y)} F(x,y) dx dy \\
 &= \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y) dx dy.
 \end{aligned}$$

Note: the text refers to regions D of this sort as Type II regions.

Note: A double integral is a number associated w/ a fun f & a region D , and this number exists & has a meaning independent of any particular method of computing it.

An iterated integral is a double integral plus a built in computational procedure.

Ex 1: $\iint_D xy^2 dA$ where D is enclosed by $x=0$ & $x=\sqrt{1-y^2}$



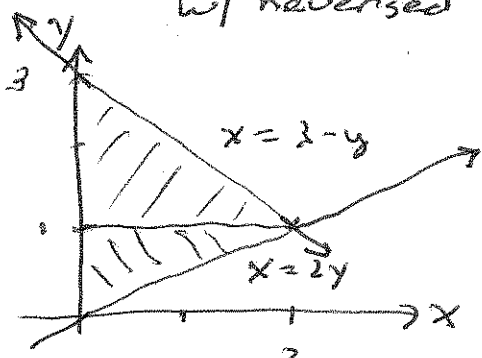
$$D = \{(x,y) \mid -1 \leq y \leq 1 \text{ AND } 0 \leq x \leq \sqrt{1-y^2}\}$$

$$\begin{aligned}
 \hookrightarrow \iint_D xy^2 dA &= \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} xy^2 dx dy \\
 &= \int_{-1}^1 \left[\frac{x^2}{2} y^2 \right]_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy \\
 &= \int_{-1}^1 \frac{1-y^2}{2} \cdot y^2 dy \\
 &= \left[\frac{y^3}{6} - \frac{y^5}{10} \right]_{-1}^1 \\
 &= \left(\frac{1}{6} - \frac{1}{10} \right) - \left(-\frac{1}{6} + \frac{1}{10} \right) \\
 &= \frac{1}{3} - \frac{2}{10} = \frac{4}{30} = \frac{2}{15}
 \end{aligned}$$

Ex 2: In evaluating a double integral over D , the following was found.

$$\iint_D f(x,y) dA = \int_0^1 \int_0^{2y} f(x,y) dx dy + \int_1^2 \int_0^{2-y} f(x,y) dx dy.$$

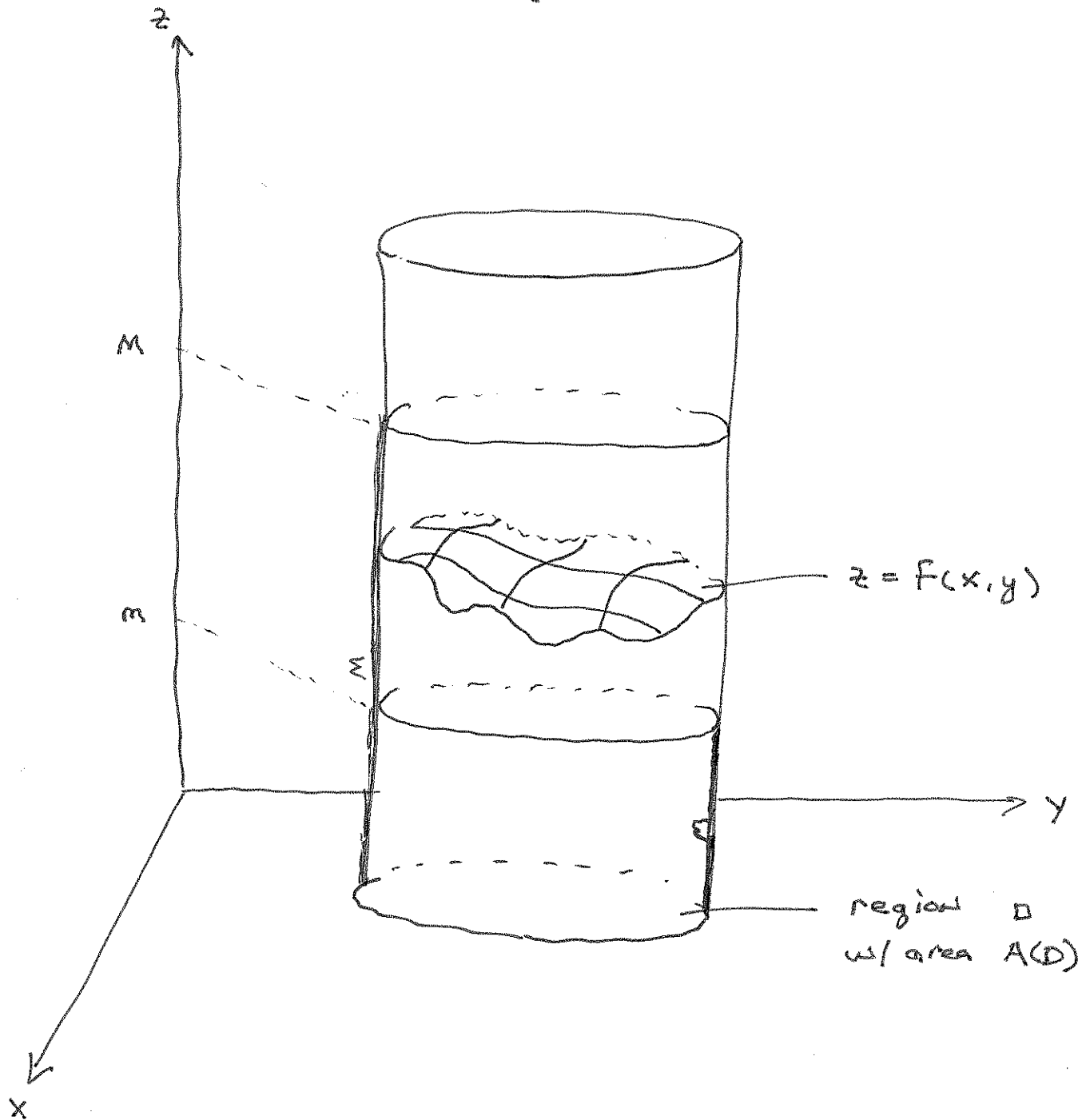
Sketch D & express the double integral w/ reversed order of integration.



$$\iint_D f(x,y) dA = \int_0^2 \int_{\frac{1}{2}x}^{2-x} f(x,y) dy dx$$

property of the double integral

□ IF $m \leq f(x,y) \leq M$ in D , then
 $m A(D) \leq \iint_D f(x,y) dA \leq M A(D)$



additional double integrals.

15.2
5/5

ex 3: $\iint_D x\sqrt{y^2-x^2} dA$ $D = \{(x,y) \mid 0 \leq y \leq 1 \text{ \& } 0 \leq x \leq y\}$

$$= \int_0^1 \int_{x=0}^{x=y} x\sqrt{y^2-x^2} dx dy$$

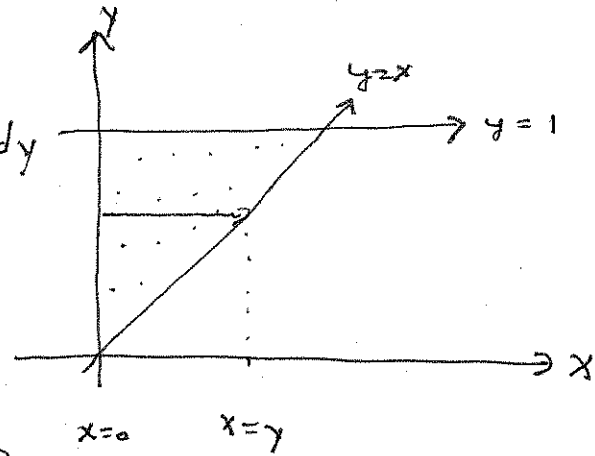
$$= -\frac{1}{2} \int_0^1 \int_{u=y^2}^{u=0} \sqrt{u} du dy$$

$$= -\frac{1}{2} \int_0^1 \left[\frac{2}{3} u^{3/2} \right]_{u=y^2}^{u=0} dy \quad u = y^2 - x^2$$

$$= -\frac{1}{2} \cdot \frac{2}{3} \int_0^1 -y^3 dy \quad \frac{du}{-2} = x dx$$

$$= \frac{1}{3} \cdot \frac{1}{4} [y^4]_0^1$$

$$= \frac{1}{12}$$



ex 4: Find the volume of the region bounded by the planes $z=x$; $y=x$; $x+y=2$; $z=0$.

$$V = \int_0^1 \int_{y=x}^{y=2-x} (x-0) dy dx$$

