What is mathematics, Really?
by Reuben Hersh
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Bring 2 typed copies
1. summary statement
2. Personal Insight
3. A question.

Dialogue with Laura

I was pecking at my word processor when twelve-year-old Laura came over.

L: What are you doing?

R: It's philosophy of mathematics.

L: What's that about?

R: What's the biggest number?

L: There isn't any!

R: Why not?

L: There just isn't! How could there be?

R: Very good. Then how many numbers must there be?

L: Infinite many, I guess.

R: Yes. And where are they all?

L: Where?

R: That's right. Where?

L: I don't know. Nowhere. In people's heads, I guess.

R: How many numbers are in your head, do you suppose?

L: I think a few million billion trillion.

R: Then maybe everybody has a few million billion trillion or so?

L: Probably they do.

R: How many people could there be living on this planet right now?

L: Don't know. Probably billions.

R: Right. Less than ten billion, would you say?

L: Okay.

R: If each one has a million billion trillion numbers or less in her head, we can count up all their numbers by multiplying ten billion times a million billion trillion. Is that right?

L: Sounds right to me.

R: Would that number be infinite?

L: Would be pretty close.

R: Then it would be the largest number, wouldn't it?

L: Wait a minute. You just asked me that, and I said there couldn't be a largest number!

R: So there actually has to be a number bigger than the biggest number in anybody's head?

L: Right.

R: Where is that number, if not in anybody's head?

L: Maybe it's how many grains of sand in the whole universe.

R: No. The smallest things in the universe are supposed to be electrons. Much smaller than grains of sand. Cosmologists say the number of electrons in the universe is less than a 1 with 23* zeroes after it. Now, ten billion times a million billion trillion is a 1 with

$$1 + 9 + 6 + 9 + 12$$

zeroes after it. That's a 1 with 37 zeroes after it, which is a hundred trillion times as much as a one with 23 zeroes it, which is more than the number of elementary particles in the universe, according to cosmologists.

L: Cosmologists are people who figure out stuff about the cosmos?

R: Right.

L: Awesome!

R: So there are way more numbers than there are elementary particles in the whole cosmos.

L: Pretty weird!

R: Never mind "where." Let's talk about "when." How long do you suppose numbers have been around?

L: A real long time.

R: Have they told you in school about the Big Bang?

L: I heard about it. It was like fifteen billion years ago. When the cosmos began.

R: Do you think there were numbers at the time of the big bang?

* Friends tell me 23 is way, way, too small. My apologies to all, especially Laura.

L: Yes, I think so. Just to count what was going on, you know.

R: And before that? Were there any numbers before the Big Bang? Even little ones, like 1, 2, 3?

L: Numbers before there was a universe?

R: What do you think?

L: Seems like there couldn't be anything before there was anything, you know what I mean? Yet it seems like there should always be numbers, even if there isn't a universe.

R: Take that number you just came up with, 1 with 37 zeroes after it, and call it a name, any name.

L: How about 'gazillion'?

R: Good. Can you imagine a gazillion of anything?

L: Heck no.

R: Could you or anyone you know ever count that high?

L: No. I bet a computer could.

R: No. The earth and the sun will vanish before the fastest computer ever built could count that high.

L: Wow!

R: Now, what is a gazillion and a gazillion?

L: Two gazillion. How easy!

R: How do you know?

L: Because one anything and another anything is two anything, no matter what.

R: How about one little mousie and one fierce tomcat? Or one female rabbit and one male rabbit?

L: You're kidding! That's not math, that's biology.

R: You never saw a gazillion or anything near it. How do

you know gazillions aren't like rabbits?

L: Numbers can't be like rabbits.

R: If I take a gazillion and add one, what do I get?

L: A gazillion and one, just like a thousand and one or a million and one.

R: Could there be some other number between a gazillion and a gazillion and one?

L: No, because a gazillion and one is the next number after a gazillion.

R: But how do you know when you get up that high the numbers don't crowd together and sneak in between each other?

L: They can't, they've got to go in steps, one step at a time.

R: But how do you know what they do way far out where

you've never been?

L: Come on, you've got to be joking.

R: Maybe. What color is this pencil?

L: Blue.

R: Sure?

L: Sure I'm sure.

R: Maybe the light out here is peculiar and makes colors look wrong? Maybe in a different light you'd see a different color?

L: I don't think so.

R: No, you don't. But are you absolutely sure it's absolutely impossible?

L: No, not absolutely, I guess.

R: You've heard of being color blind, haven't you?

L: Yes, I have.

R: Could it be possible for a person to get some eye disease and become color blind without knowing it?

L: I don't know. Maybe it could be possible.

R: Could that person think this pencil was blue, when actually it's orange, because they had become color blind without knowing it?

L: Maybe they could. What of it? Who cares?

R: You see a blue pencil, but you aren't 100% sure it's really blue, only almost sure. Right?

L: Sure. Right.

R: Now, how about a gazillion and a gazillion equals two gazillion? Are you absolutely sure of that?

L: Yes I am.

R: No way that could be wrong?

L: No way.

R: You've never seen a gazillion. Yet you're more sure about gazillions than you are about pencils that you can see and touch and taste and smell. How do you get to know so much about gazillions?

L: Is that philosophy of mathematics?

R: That's the beginning of it.