100	90's	80's	70's	60's	< 60
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Test 2
Dusty Wilson
Math 152

high = 100%
$\bar{x} = 74.7\%$
med = 72.6%

No work = no credit

No Symbolic Calculators

Name: Key

Why are numbers beautiful? It's like asking why is Beethoven's Ninth Symphony beautiful. If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is.

Paul Erdös (1913 - 1996)
Hungarian mathematician

Warm-ups (1 pt each): $-3^2 =$ -9 $\qquad -3^0 =$ -1 $\qquad \sqrt{(-3)^2} =$ 3

1.) (1 pt) According to Erdös, how do we know that numbers are beautiful?

Some things we must know with our hearts.

2.) (10 pts) Integrate $I = \int e^{\sqrt{x}}\,dx$

Let $w = \sqrt{x}$

$dw = \frac{1}{2\sqrt{x}}\,dx$

or $2w\,dw = dx$

$$= 2\int w e^{w}\,dw$$

Let $u = w \qquad v = e^{w}$

$du = dw \qquad dv = e^{w}\,dw$

$$= 2\left[we^{w} - \int e^{w}\,dw\right]$$

$$= 2\left[\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}}\right] + C$$

3.) (10 pts) Integrate $I = \int \frac{2x+1}{x^2 - 7x + 12}\,dx$

$$= \int \frac{9}{x-4} - \frac{7}{x-3}\,dx$$

$$= 9\ln|x-4| - 7\ln|x-3| + C$$

scratch

$$\frac{2x+1}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$$

$$\Rightarrow 2x + 1 = A(x-3) + B(x-4)$$

$$= Ax - 3A + Bx - 4B$$

$$\Rightarrow \left[\begin{array}{cc|c} -3 & -4 & 1 \\ 1 & 1 & 2 \end{array}\right]$$

Ans $A = 9 \qquad B = -7$

4.) (10 pts) Integrate $I = \int \frac{\sin\theta}{8\cos^3\theta} d\theta$

Let $u = \cos\theta$

$du = -\sin\theta \, d\theta$

$$= -\frac{1}{8} \int \frac{du}{u^3}$$

$$= -\frac{1}{8} \cdot \frac{1}{-2} u^{-2} + c$$

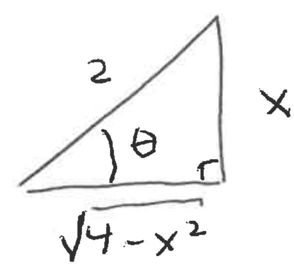
$$= \frac{1}{16} \sec^2\theta + c$$

5.) (10 pts) Integrate $I = \int \frac{dx}{(4-x^2)^{3/2}}$

Let $x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

$$= \int \frac{2\cos\theta}{(4-4\sin^2\theta)^{3/2}} d\theta$$



$$= \int \frac{2\cos\theta}{8(1-\sin^2\theta)^{3/2}} d\theta$$

$$= \frac{1}{4} \int \frac{\cos\theta}{\cos^3\theta} d\theta$$

$$= \frac{1}{4} \int \sec^2\theta \, d\theta$$

$$= \frac{1}{4} \tan\theta + c$$

$$= \frac{1}{4} \cdot \frac{x}{\sqrt{4-x^2}} + c$$

6.) (10 pts, -2pts) A tank contains 100 L of pure water. Brine that contains 0.1 kg of salt per liter enters this tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. If $y(t)$ is the quantity of salt in kg dissolved in the tank at time *t*, then *y* satisfies

a.) $\frac{dy}{dt}=1-\frac{y}{10}, y(0)=0.1$

b.) $\boxed{\frac{dy}{dt}=1-\frac{y}{10}, y(0)=0}$

c.) $\frac{dy}{dt}=-\frac{y}{10}, y(0)=0$

d.) $\frac{dy}{dt}=-\frac{y}{100}, y(0)=0.1$

e.) $\frac{dy}{dt}=10-\frac{y}{100}, y(0)=0$

RATE In

$0.1\ \frac{kg}{L}$ @ 10 L/min

RATE OUT $\frac{y(t)}{100}\frac{kg}{L}\cdot 10\frac{L}{m}$

Let $y(t)$ = kg of salt after t min.

since the tank begins filled w/ pure water: $y(0)=0$

$$\frac{dy}{dt} = 0.1(10) - \frac{y(t)}{100}\cdot 10 = 1 - \frac{y(t)}{10}$$

7.) (10 pts) Complete 1 of the following 3. Clearly indicate your results. Use scratch paper as needed.

Find the general solution of $\frac{dy}{dx}=(2+3x^2)(y^2+1)$

$$\frac{dy}{y^2+1} = (2+3x^2)\,dx$$

$$\Rightarrow \int \frac{dy}{y^2+1} = \int (2+3x^2)\,dx$$

$$\Rightarrow \arctan(y) = 2x + x^3 + C$$

$$\Rightarrow y = \tan(2x + x^3 + C).$$

Integrate $I=\int e^x \sin(x)\,dx$

$u = e^x$; $v = -\cos x$

$du = e^x dx$; $dv = \sin x\,dx$

$$\Rightarrow I = -e^x\cos x + \int e^x \cos x\,dx$$

$u = e^x$; $v = \sin x$

$du = e^x dx$; $dv = \cos x\,dx$

$$\Rightarrow I = -e^x\cos x + e^x \sin x - \underbrace{\int e^x \sin x\,dx}$$

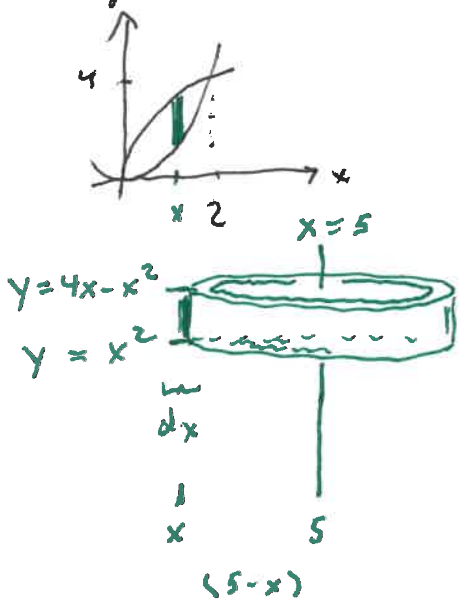
$$\Rightarrow I = \frac{1}{2}e^x(\sin x - \cos x) + C$$

Find the average value of $f(x)=\sin(x)$ on $[0,\pi]$

$$f_{ave} = \frac{1}{\pi - 0}\int_0^{\pi} \sin x\,dx$$

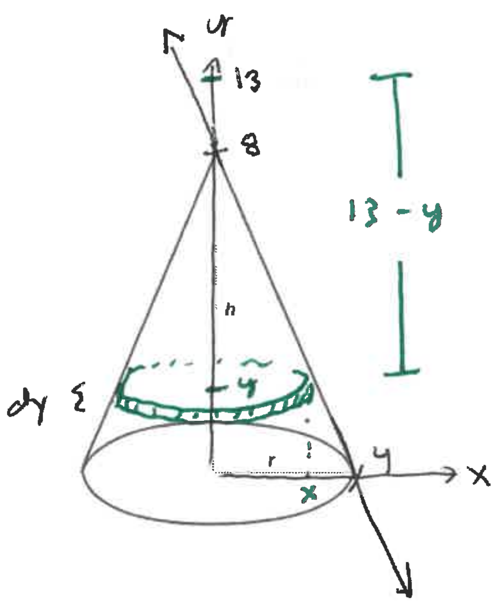
$$= \frac{2}{\pi}$$

8.) (10 pts) Set up an integral to find the volume of the solid formed when the region bounded between $y = x^2$ and $y = 4x - x^2$ is rotated about the line $x = 5$. Do not evaluate. (Volume: $V = \frac{64}{3}\pi$)



$$V = 2\pi \int_0^2 (5-x)(4x-2x^2)\,dx$$

9.) (10 pts) A cone shaped tank is filled completely with water. The tank has a height of 8 meters and a radius of 4 meters. Set up an integral to find the work required to pump the water out of the tank to a height 5 meters above the tank? Do not evaluate. (Work: $W \approx 15{,}000{,}000$ N-m)



$$d\,\text{Area} = \pi x^2$$

$$d\,\text{volume} = \pi x^2\,dy$$

$$d\,\text{mass} = 1000\pi x^2\,dy$$

$$d\,\text{force} = 1000(9.8)\pi x^2\,dy$$

$$d\,\text{work} = 1000(9.8)\pi\, x^2 (13-y)\,dy$$

$$\text{Total work} = \int_0^8 1000(9.8)\pi \left[r\left(1-\frac{y}{h}\right)\right]^2 (13-y)\,dy$$

$$= \int_0^8 1000(9.8)\pi \left(4 - \tfrac{1}{2}y\right)^2 (13-y)\,dy$$

$$y = -\frac{h}{r}x + h$$

$$\Rightarrow y - h = -\frac{h}{r}x$$

$$\Rightarrow x = \frac{r}{h}(h-y) = r\left(1-\frac{y}{h}\right)$$