100	90's	80's	70's	60's	<60
0	2	8	6	1	3

high = 92.6%
$\bar{x}$ = 76.4%
med = 79.3%

6:46
7:06

Test 1
Dusty Wilson
Math 152

Name: key

A mathematician is a machine for turning coffee into theorems.

Paul Erdös (1913 - 1996)
Hungarian mathematician

No work = no credit

No Symbolic Calculators

Warm-ups (1 pt each): $-2^2 =$ -4 $\quad -2^0 =$ -1 $\quad \sqrt{(-2)^2} =$ 2

1.) (1 pt) According to Erdös (see quote under your name), what fuels the mathematicians theoretical fervor?

coffee provides the power!

2.) (10 pts) Integrate $I = \int \left(3x^4 + \frac{7}{x} - 5\sqrt{x} + 2e^x \right) dx$

$$= \int (3x^4 + 7x^{-1} - 5x^{1/2} + 2e^x)\,dx$$

$$= \frac{3}{5}x^5 + 7\ln|x| - \frac{10}{3}x^{3/2} + 2e^x + C$$

3.) (10 pts) Approximate the definite integral $\int_0^1 \underbrace{\frac{4}{1+x^2}}_{f(x)} dx$ using Simpson's Rule with $n = 4$. Give your answer to ~~four~~ five decimal places and show enough work to convince me you know what you are doing.

$$\int_0^1 f(x)\,dx \approx \frac{0.25}{3}\left(1f(0) + 4f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 4f(\tfrac{3}{4}) + 1f(1)\right)$$

$$= 3.14157$$

The next two questions are about Riemann Sums and the Definition of the Definite Integral.

4.) (10 pts) Use the Definition of the Definite Integral to write the $\int_{-2}^{1}(4x^5-7)\,dx$ as the limit of Riemann Sums. Do not evaluate.

$$\Delta x = \frac{3}{n}$$

$$x_i = -2 + \frac{3i}{n}$$

$$\int_{-2}^{1}(4x^5-7)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left[4\left(-2+\frac{3i}{n}\right)^5-7\right]\frac{3}{n}$$

no penalty

5.) (10 pts) Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n}\left[4\left(2+\frac{3i}{n}\right)+3\right]\frac{3}{n}$ $= \lim_{n\to\infty}\sum_{i=1}^{n}\left(8+\frac{12i}{n}+3\right)\frac{3}{n}$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{33}{n}+\frac{36i}{n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{33}{n}\cdot n+\frac{36}{n^2}\frac{n(n+1)}{2}\right)$$

$$= 33 + 18$$

$$= 51$$

6.) (10 pts) Integrate $I = \int_0^1 \frac{x^3}{\sqrt{1+x^4}}\,dx$

Let $u = 1 + x^4$ $\quad u(0) = 1$

$\frac{du}{4} = x^3\,dx$ $\quad u(1) = 2$

$$= \int_1^2 \frac{1}{4} \cdot \frac{1}{\sqrt{u}}\,du$$

$$= \int_1^2 \frac{1}{4} u^{-1/2}\,du$$

$$= \frac{1}{4}\left[2u^{1/2}\right]_1^2$$

$$= \frac{1}{2}(\sqrt{2} - 1)$$

$\frac{7}{10}$ if lim all wrong.

$\frac{8}{10}$ if not changed but sub back

$\frac{9}{10}$ if changed but missed once

7.) (10 pts) Evaluate $\frac{d}{dx}\int_{\sin(x)}^1 e^{t^2}\,dt$

Let $A(x) = \int_1^x e^{t^2}\,dt$ (cumulative area fct)

then $A'(x) = e^{x^2}$.

now $\frac{d}{dx}\int_{\sin x}^1 e^{t^2}\,dt = \frac{d}{dx} - A(\sin x)$

$$= -A'(\sin x) \cdot \cos x$$

$$= -\cos x\, e^{\sin^2 x}$$

7/10 if no work

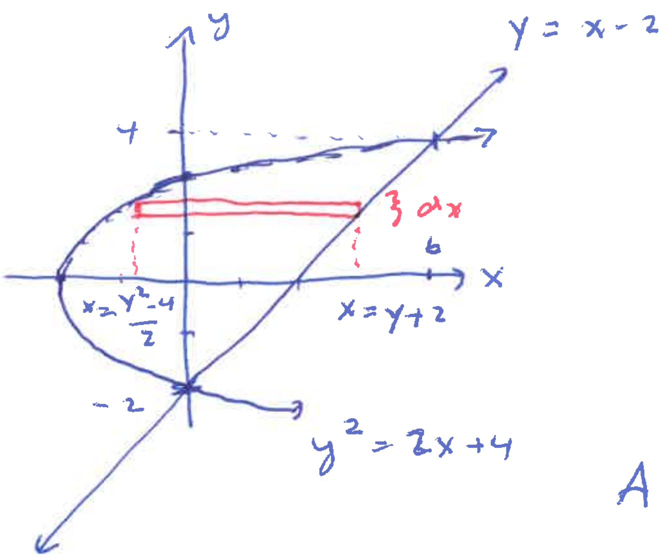
8/10 if some mention of FTofC 1

4.) (10 pts) Write the Fundamental Theorem of Calculus (part 2).

If f is continuous on [a, b], then … $\int_a^b f(x)\,dx = F(b) - F(a)$

where $F' = f$, that is F is any antiderivative of f.

8.) (10 pts) <u>Set up</u> an integral to represent the area enclosed by the line $y = x - 2$ and the parabola $y^2 = 2x + 4$ (Area: $A =$ 18)



solve: $(x-2)^2 = 2x+4$

$\Rightarrow x^2 - 4x + 4 = 2x + 4$

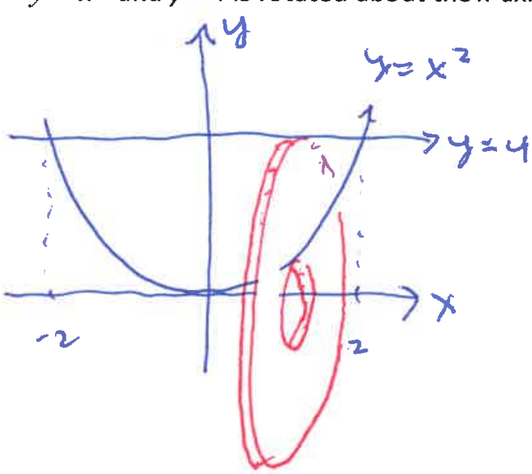
$\Rightarrow x^2 - 6x = 0$

$\Rightarrow x(x-6) = 0$

$\Rightarrow x = 0$ OR $x = 6$

$$A = \int_{-2}^{4} \left[(y+2) - \left(\frac{y^2-4}{2} \right) \right] dy$$

9.) (10 pts) <u>Set up</u> an integral to find the volume of the solid formed when the region bounded between $y = x^2$ and $y = 4$ is rotated about the x-axis. <u>Do not evaluate</u>. (Volume: $V = \frac{256}{5}\pi$)



$$V = \int_{-2}^{2} \left[\pi \cdot 4^2 - \pi (x^2)^2 \right] dx$$

$$= 2\pi \int_{0}^{2} (16 - x^4) dx$$