Group Quiz 4
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Math 152 – Spring 2019

Name: Key.

No work = no credit

1.) $I=\int_1^2 \frac{98+11x-x^2}{49x+14x^2+x^3}\,dx$

$= \int_1^2 \frac{2}{x} - \frac{3}{x+7} + \frac{4}{(x+7)^2}\,dx$

$= \left[2\ln|x| - 3\ln|x+7| - \frac{4}{x+7}\right]_1^2$

$= \left(2\ln 2 - 3\ln 9 - \frac{4}{9} - 0 + 3\ln 8 + \frac{4}{8}\right)$

$= \frac{1}{18} + \ln\frac{2048}{729}$

≈ 1.0885

Scratch

$49x+14x^2+x^3$

$= x(49+14x+x^2)$

$= x(x+7)^2$

> The integral $\int \sec(x)\,dx$ was first computed numerically in the 1590's by the English mathematician Edward Wright, decades before the invention of calculus. Although he did not invent the concept of an integral, Wright realized that the sums that approximate the integral hold the key to understanding the Mercator map projection, of great importance in sea navigation because it enabled sailors to reach their destinations along lines of fixed compass direction. The formula for the integral was first proved by James Gregory in 1668.

2.) A culinary crime is in the works. A vat of 500 gallons of dark (72% cocoa) chocolate (in liquid form) is polluted by a steady stream of 5 gal/min liquid milk chocolate (10% cocoa). The chocolate is being constantly stirred and the contaminated concoction overflows the tank creating a huge mess! An hour after the sticky influx begins; a confectioner notices the travesty and stops the process. What is the cocoa content in the vat of "chocolate" that remains?

#1 scratch continued.

$$\frac{98+11x-x^2}{49x+14x^2+x^3} = \frac{98+11x-x^2}{x(x+7)^2}$$

$$= \frac{A}{x} + \frac{B}{x+7} + \frac{C}{(x+7)^2}$$

$$\Rightarrow 98+11x-x^2 = A(x+7)^2 + Bx(x+7) + Cx$$

$$= Ax^2 + 14Ax + 49A + Bx^2 + 7Bx + Cx$$

Set up a system,

1's : $49A = 98$

x's $14Ax + 7Bx + Cx = 11x$

x^2's $Ax^2 + Bx^2 = -x^2$

Form a matrix

$$\begin{bmatrix} 49 & 0 & 0 & 98 \\ 14 & 7 & 1 & 11 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

row $\downarrow$ reduce

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

So $A = 2$, $B = -3$, and $C = 4$

chocolate

$C(t) = \#$ of gals of choc. in the tank after t min.

$$\frac{dc}{dt} = (\text{rate in}) - (\text{rate out})$$

$$= 5 \cdot 0.1 \frac{\text{gal}}{\text{min}} - \frac{5c(t)}{500} \frac{\text{gal}}{\text{min}}$$

$$\Rightarrow \frac{dc}{dt} = 0.5 - 0.01c$$

$$\Rightarrow dc = (0.5 - 0.01c)\, dt$$

$$\Rightarrow \int \frac{dc}{0.5 - 0.01c} = \int dt$$

$$\Rightarrow -100 \ln|0.5 - 0.01c| = t + C_1$$

$$\Rightarrow \ln|0.5 - 0.01c| = \frac{t}{-100} + C_1$$

$$\Rightarrow 0.5 - 0.01c = ke^{-\frac{1}{100}t}$$

$$\Rightarrow -0.01c = -0.5 + ke^{-\frac{1}{100}t}$$

$$\Rightarrow c = 50 + ke^{-\frac{1}{100}t}$$

we know $c(0) = 0.72(50) = 360$ gal. $\Rightarrow k = 310$

$$c(t) = 50 + 310e^{-\frac{1}{100}t}$$

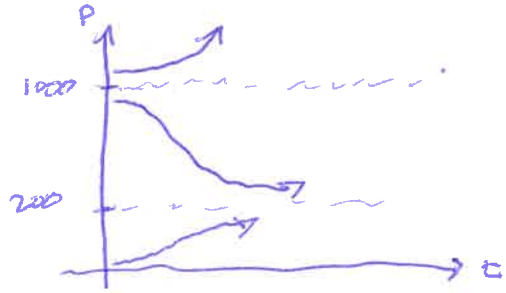
$$c(\ ) = 50 + 310e^{-\frac{6}{10}} \approx 220{,}132 \text{ gal} \text{ or } 44\% \text{ cocoa.}$$

3.) There is considerable evidence to support the theory that for some species there is a minimum population m such that the species will become extinct if the size of the population falls below m. This is captured in the modified logistic equation $\frac{dP}{dt} = kP\left(1-\frac{P}{K}\right)\left(1-\frac{P}{m}\right)$.

a.) For $k = 0.08$, $K = 1000$, and $m = 200$, find the equilibrium solutions.

equilibrium is where

$\frac{dP}{dt} = 0$ or $P = 0, 1000, 200$



b.) Solve the differential equation explicitly given $P_0 = 500$.

$$\frac{dP}{dt} = \frac{k}{Km} P(K-P)(m-P)$$

Scratch

$$\frac{1}{P(K-P)(m-P)} = \frac{A}{P} + \frac{B}{K-P} + \frac{C}{m-P}$$

$$\Rightarrow 1 = A(K-P)(m-P) + BP(m-P) + CP(K-P)$$

$$\Rightarrow \int \frac{dP}{P(K-P)(m-P)} = \int \frac{k}{Km}\, dt$$

$$\Leftrightarrow \int \frac{\frac{1}{Km}}{P} + \frac{\frac{1}{K(m-K)}}{K-P} + \frac{\frac{1}{m(K-m)}}{m-P}\, dP = \frac{k}{Km} t + C_1$$

$$\Rightarrow \frac{1}{Km}\ln|P| - \frac{1}{K(m-K)}\ln|K-P| - \frac{1}{m(K-m)}\ln|m-P| = \frac{k}{Km} t + C_1$$

$$\Rightarrow \ln|P| - \frac{m}{m-K}\ln|K-P| - \frac{K}{K-m}\ln|m-P| = kt + C_2$$

$$\Rightarrow \ln|P| + \frac{1}{K-m}\left(-K\ln|m-P| + m\ln|K-P|\right) = kt + C_2$$

$$\Rightarrow \ln\left|P\left[(m-P)^{-K}(K-P)^{m}\right]^{\frac{1}{K-m}}\right| = kt + C_2$$

$$\Rightarrow P|m-P|^{\frac{-K}{K-m}}|K-P|^{\frac{m}{K-m}} = C_3 e^{kt}$$ (General solution).

solve the IVP

$P(0) = 500$

$$500(300)^{-5/4}(500)^{1/4} = C_3$$

or $C_4 = \left(\frac{500}{300}\right)^5 = 12.86$

AND $P^4|200-P|^{-5}|1000-P| = 12.86e^{0.32t}$