Group Quiz 2
Dusty Wilson
Math 152

Name: key.

No work = no credit

1.) Evaluate $I = \int_0^{\frac{\pi}{6}} \cos(3x)\sin(\sin(3x))\,dx$

Let $u = \sin(3x)$

$\frac{du}{3} = \cos(3x)\,dx$

$u(0) = 0$

$u(\frac{\pi}{6}) = 1$

$$= \frac{1}{3}\int_0^1 \sin u \, du$$

$$= \frac{1}{3}\left[-\cos u\right]_0^1$$

$$= \frac{1}{3}\Big(-\cos(1) + \underbrace{\cos(0)}_{1}\Big)$$

$$= \frac{1}{3}\left(1 - \cos(1)\right)$$

2.) If $F(x) = \int_x^2 f(t)\,dt$ where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\,du$, find $F''(2)$

$$F''(x) = \frac{d^2}{dx^2}\int_x^2 \int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\,du\,dt$$

$$= -\frac{d}{dx}\int_1^{x^2} \frac{\sqrt{1+u^4}}{u}\,du$$

$\leftarrow$ let $A(x) = \int_1^x \frac{\sqrt{1+u^4}}{u}\,du$

$$= -\frac{d}{dx} A(x^2)$$

$A'(x) = \frac{\sqrt{1+x^4}}{x}$

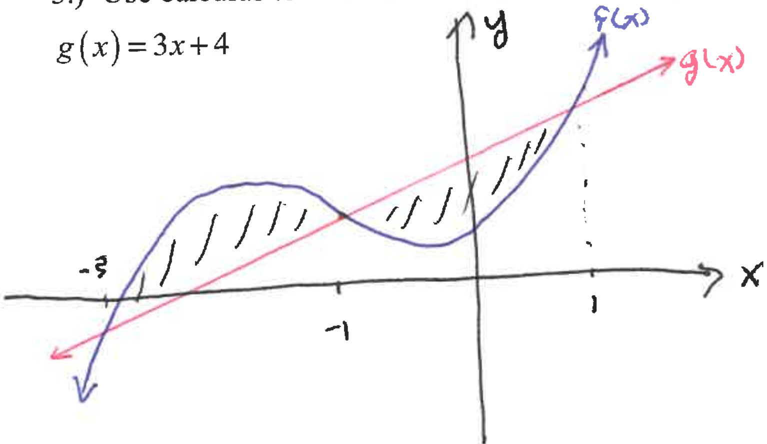
$$= -2x\,A'(x^2)$$

$$= \frac{-2x\sqrt{1+x^8}}{x^2}$$

$\rightarrow$ AND $F''(2) = \frac{-4\sqrt{257}}{4}$

$= -\sqrt{257}$

3.) Use calculus to find the exact area between the curves $f(x) = x^3 + 3x^2 + 2x + 1$ and $g(x) = 3x + 4$



$$\text{Area} = \int_{-3}^{-1} (f(x) - g(x))\,dx + \int_{-1}^{1} (g(x) - f(x))\,dx$$

$$= \int_{-3}^{-1} (x^3 + 3x^2 - x - 3)\,dx + \int_{-1}^{1} (-x^3 - 3x^2 + x + 3)\,dx$$

$$= \left[\frac{x^4}{4} + x^3 - \frac{x^2}{2} - 3x\right]_{-3}^{-1} + \left[-\frac{x^4}{4} - x^3 + \frac{x^2}{2} + 3x\right]_{-1}^{1}$$

$$= \left[\left(\frac{1}{4} - 1 - \frac{1}{2} + 3\right) - \left(\frac{81}{4} - 27 - \frac{9}{2} + 9\right)\right] +$$

$$\left[\left(\cancel{-\frac{1}{4}} - 1 + \cancel{\frac{1}{2}} + 3\right) - \left(\cancel{-\frac{1}{4}} + 1 + \cancel{\frac{1}{2}} - 3\right)\right]$$

$$= 8$$

Alternate solution #1

If $F(x) = \int_x^2 f(t)\,dt$

$$= -\int_2^x f(t)\,dt$$

$$\Rightarrow F'(x) = -f(x) \qquad \text{(by FToC 1)}$$

$$= -\int_1^{x^2} \frac{\sqrt{1+u^4}}{u}\,du$$

$$\Rightarrow F''(x) = -\frac{d}{dx}\int_1^{x^2} \frac{\sqrt{1+u^4}}{u}\,du \qquad \text{Let } w = x^2$$

$$= -\frac{d}{dw}\int_1^{w} \frac{\sqrt{1+u^4}}{u}\,du \; \frac{dw}{dx}$$

$$= -\frac{\sqrt{1+w^4}}{w}\,\frac{dw}{dx} \qquad \text{(by FToC 1)}$$

$$= -\left.\frac{\sqrt{1+x^8}}{x^2}\cdot 2x\right|_{x=2} \qquad -\sqrt{257}$$

Alternate solution #2

If $F(x) = \int_x^2 f(t)\,dt$

$$= -\int_2^x f(t)\,dt$$

$\Rightarrow F'(x) = -f(x)$ (by FToC 1)

Now let $A(x) = \int_1^x \frac{\sqrt{1+u^4}}{u}\,du$ This is the cumulative area fct.

$\Rightarrow A'(x) = \frac{\sqrt{1+x^4}}{x}$ (by FToC 1)

We are now ready to go back to F and f

$$F''(x) = -f'(x) = -\frac{d}{dx} A(x^2)$$

$$= -A'(x^2)\cdot 2x$$

$$= -\frac{\sqrt{1+x^8}}{x^2}\cdot 2x \Big|_{x=2} - \sqrt{257}$$