Group Quiz 1
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Math 152

Name: Key.

No work = no credit

(1.1) Use the definition of the definite integral to evaluate $\int_1^7 \left(x^2 - 3x + 4\right) dx$ exactly.

$\Delta x = \frac{6}{n}$

$x_i = 1 + \frac{6i}{n}$

$$\int_1^7 x^2 - 3x + 4\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1 + \tfrac{6i}{n}\right)^2 - 3\left(1 + \tfrac{6i}{n}\right) + 4\right] \tfrac{6}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 + \tfrac{12i}{n} + \tfrac{36i^2}{n^2} - 3 - \tfrac{18i}{n} + 4\right] \tfrac{6}{n}$$

$$2 - \frac{6i}{n} + \frac{36i^2}{n^2}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{12}{n} - \frac{36i}{n^2} + \frac{216i^2}{n^3}\right)$$

$$= \lim_{n\to\infty} \left(12 - \frac{36}{n^2} \cdot \frac{n(n+1)}{2} + \frac{216}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right)$$

$$= 12 - 18 + \frac{216}{3}$$

$$= 66.$$

(1.2) Consider $\int_0^{\pi/2} \sin(2x)\,dx$. Suppose you intend to approximate this definite integral using Simpson's Rule. You need your results to have error $|E_S| \le 0.00005$. Find n and S_n.

$f^{(4)}(x) = 16\sin(2x) \Rightarrow k = 16.$

$\Rightarrow \dfrac{k\cdot(\pi/2)^5}{180\,n^4} \le 0.00005 \Rightarrow \sqrt[4]{\dfrac{16(\pi/2)^5}{180(0.00005)}} \le n = 6.42$

$S_{10} = \frac{1}{3}\left(f(0) + 4f(\tfrac{1\pi}{16}) + 2f(\tfrac{2\pi}{16}) + \dots + 4f(\tfrac{7\pi}{16}) + f(\tfrac{8\pi}{16})\right)\frac{\pi}{16}$

$= 1.00013$

(which is w/in the error bound of the exact sum 1).

(1.3) This morning I looked up high school dropout rates in King County over the last few years. If t is given in school years ending since 2000, we can model the dropout rate (in students per year) over the last six years with $D(t) = 29.4t^3 - 711t^2 + 5500t - 12600$. Use Riemann sums to approximate the total number of dropouts between 2005 and 2010. Compare this to the actual total of 4749 dropouts and explain the difference.

t	5	6	7	8	9	10
$D(t)$	800	1154.4	1145.2	948	741.6	700

$L_5 = 4789.2$

$R_5 = 4689.2$

$T_5 = 4739.2$

Can't do S_5

These are only approximations based on a model.