Notetaker's name ____________ Class ____________ Date May 5. 09

Section 9.4

Population Models

1) $P' = kP$

2) $P' = kP\left(1 - \frac{P}{K}\right)$

Solve: $P' = kP$

$\Rightarrow \frac{dP}{dt} = kP$

$\Rightarrow \frac{dP}{P} = k\,dt$

$\Rightarrow \int \frac{dP}{P} = \int k\,dt$

$\ln|P| = kt + C_1$

$|P| = e^{kt + C_1}$

$|P| = C_2 e^{kt}$

$\Rightarrow P = \pm C_2 e^{kt}$

what is C_2

Suppose $P(0) = P_0$

$P(0) = P_0 = C_2 \underbrace{e^0}_{1}$

$\Rightarrow C_2 = P_0$

The solution to $P' = kP$

is $P(t) = P_0 e^{kt}$

k = initial growth rate

P_0 = initial Population.

Ex:2 Biologists stock a lake w/ 400 fish and estimate that the carrying capacity of the lake is 10,000. The number of fish tripled the 1st year. a) Find a model $P(t)$ for the population after t years. ← ×3.

$\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$

K is carrying capacity which is 10000.

k is initial growth rate.

$\Rightarrow$ Solve $\frac{dP}{dt} = kP\left(1 - \frac{P}{10000}\right)$

Notetaker's name ______ Class ______ Date ______

$$\int \frac{dP}{P\left(1-\frac{P}{10000}\right)} = \int k\,dt$$

Side problem: Partial Fractions.

$$\int \frac{dP}{P\left(1-\frac{P}{10000}\right)} = \int\left(\frac{1}{P}^{\times A} + \frac{1/10000}{1-\frac{P}{10000}}^{\times B}\right) dP$$

$$\text{Integrand} = \frac{1}{P\left(1-\frac{P}{10000}\right)} = \frac{A}{P} + \frac{B}{1-\frac{P}{10000}}$$

common denominator — product — sum if we can find A and B.

$$\Rightarrow \quad 1 = A\left(1-\frac{P}{10000}\right) + B(P)$$

$$\Rightarrow \quad 1 = A - \frac{PA}{10000} + BP$$

variable is P.

combine like terms.

$$\Rightarrow \quad 1 = \underbrace{A}_{1} + P\underbrace{\left(\frac{-A}{10000} + B\right)}_{0}$$

on P on L.S.

$A = 1$

$\frac{-A}{10000} + B = 0$

$\Rightarrow \quad B = \frac{1}{10000}$

$$\int\left(\frac{1}{P} + \frac{1/10000}{1-\frac{P}{10000}}\right) dP = \ln|P| - \ln\left|1-\frac{P}{10000}\right| = \ln\left|\frac{P}{1-\frac{P}{10000}}\right|.$$

$$\frac{1}{10000}\int \frac{1}{1-\frac{P}{10000}}\,dP = -\int \frac{1}{u}\,du$$

let $u = 1 - \frac{P}{10000}$

$du = -\frac{1}{10000}\,dP$

$$\ln\left|\frac{P}{1-\frac{P}{10000}}\right| = kt + C.$$

Notetaker's name ______ Class ______ Date ______

$\ln\left|\frac{P}{1-\frac{P}{10000}}\right| = kt + C_1$ Solve for P.

$\left|\frac{P}{1-\frac{P}{10000}}\right| = e^{kt+C_1}$

$\left|\frac{P}{1-\frac{P}{10000}}\right| = C_2 e^{kt}$

$\frac{P}{1-\frac{P}{10000}} = \pm C_2 e^{kt}$ invert both sides.

$\frac{1-\frac{P}{10000}}{P} = \frac{1}{C_2 e^{kt}}$

$\Rightarrow \frac{1-\frac{P}{10000}}{P} = C_3 e^{-kt}$

$\frac{\frac{10000-P}{10000}}{P} = C_3 e^{-kt}$

$\Rightarrow \frac{10000-P}{P} = C_4 e^{-kt}$

$\frac{10000}{P} - 1 = C_4 e^{-kt}$ add 1.

$\Rightarrow \frac{10000}{P} = 1 + C_4 e^{-kt}$

$P = \frac{10000}{1+C_4 e^{-kt}}$

If $P_0 = 400$, then

$P(0) = 400 = \frac{10000}{1+C_4}$

$\Rightarrow 1 + C_4 = \frac{10000}{400}$

$1 + C_4 = 25$

$C_4 = 24$

So $P = \frac{10000}{1+24e^{-kt}}$

from the initial population of 400.

We also know $P(1) = 1200$ (400 tripled)

solve

$1200 = \frac{10000}{1+24e^{-k}}$

$1+24e^{-k} = \frac{10000}{1200}$

$e^{-k} = \frac{\frac{100}{12}-1}{24}$

So, $P(t) = \frac{10000}{1+24e^{-1.1856t}}$

$k = -\ln\left(\frac{\frac{100}{12}-1}{24}\right) = 1.1856$

Notetaker's name ______ Class ______ Date May 6.09

Section 9.4 (cont.)

from yesterday:

solve $\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$

we found $P(t) = \frac{10000}{1+24e^{-1.1255t}}$

which is of the form $P(t) = \frac{K}{1+Ae^{-kt}}$

how did we find $A = 24$

$P(0) = 400 = \frac{10000}{1+A}$

$\Rightarrow A = \frac{10000}{400} - 1$

$$\boxed{A = \frac{K}{P_0} - 1}$$

How did we find k

depends upon some other piece of info

in our case: $P(1) = 1200$.

All together:

$\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$

the solution is

$P(t) = \frac{K}{1+Ae^{-kt}}$

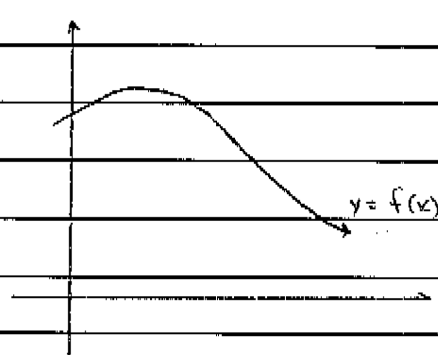
where:

$A = \frac{K}{P_0} - 1$

and k depends on some additional piece of info.

Slope fields:

EX: #3. based on (9.1 # 12)



Q: $f(x)$ is the solution to which of the following.

A) $y' = 1+xy$ >0 in I quadrant.

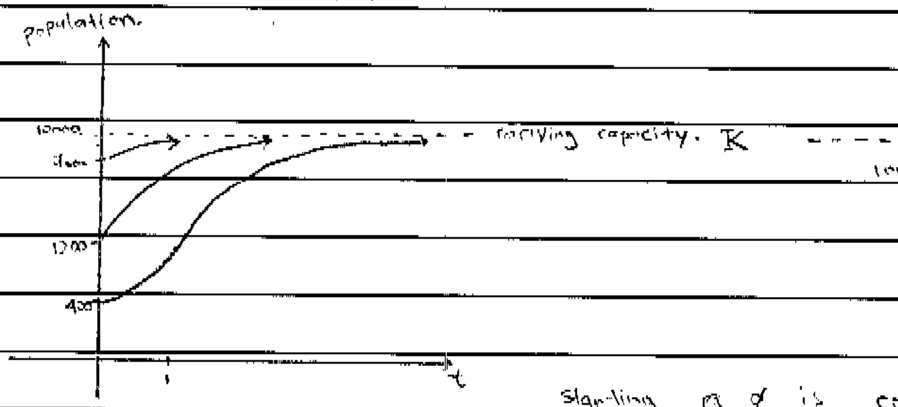
B) $y' = -2xy$ <0 in I quadrant

C) $y' = 1-2xy$ slope ok at y-int.

$\rightarrow y' = 0$ is possible in I quad

Notetaker's name ____________ Class ____________ Date ________

Ex: 2 Rev: (fish in the lake)



1000 is constant solution P=1000

stable solution.

Starting at 0 is constant solution.

unstable solution.

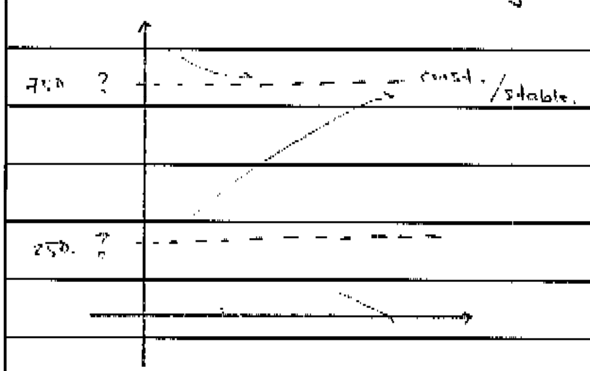
Ex: 4. lake of fish $\frac{dP}{dt} = 0.08\,P\left(1 - \frac{P}{1000}\right) - 15$

rate of change of population

initial growth rate

carrying capacity

some constant rate at which the pop is harvested.



Find when pop is constant

$\frac{dP}{dt} = 0 = 0.08\,P\left(1 - \frac{P}{1000}\right) - 15$

$\Rightarrow 0 = .08P - \frac{0.08P^2}{1000} - 15$

$\Rightarrow 0 = 80P - 0.08P^2 - 15000$

$P = 250$

$P = 750$